

Exogenous Public Spending in the Optimal Taxation Problem

Pedro Teles

Banco de Portugal, Universidade Catolica Portuguesa and CEPR;

ABSTRACT

I consider a reasonable deviation from the standard assumptions in the optimal taxation literature on public consumption and show that the results on the optimal taxes are substantially altered. In particular, ‘distortionary’ taxes may be optimal after all. This reveals that, in contrast to the common practice, the analysis of optimal taxation cannot be done independently of the analysis of the optimal public provision of goods and services.

Key words: Optimal taxation; Ramsey taxation; public consumption;

JEL classification: E61; E62; E66; H00; H21.

I. Introduction

The standard Ramsey (1927) approach to optimal taxation assumes government expenditures are exogenous.¹ This assumption is also shared by other approaches to optimal taxation, such as Mirrlees (1971). The assumption makes a lot sense. We want to be able to focus on the optimal taxes that finance expenditures, independently of the way expenditures are decided. But we also want the results on the optimal taxes to be robust to small changes in those assumptions. What should be exogenous? Should it be the level of expenditures? Should it be the share of expenditures in output, or in private consumption? Does it matter whether expenditures are transfers or public consumption?

Suppose for the moment that we take the long run share of government consumption to be a constant share of output. Suppose that we also take the simplest balanced growth model with labor only. Expenditures can be a constant share of output because they are exogenous and they grow at the same rate of exogenous technological progress, or because they are endogenous, equal to a constant, exogenous, share of output. The two assumptions which, as far as growth rates go, are indistinguishable, make a major difference for optimal tax policy. Suppose government purchases were indeed a constant share of output and the policy maker took this into account when designing optimal tax policy. Then, even if it was possible to tax lump sum, it would not be optimal to do so. The optimal labor income tax rate would be the share of government consumption in output. So if government consumption is 20% of output, the income tax rate should be 20%. Naturally, if the planner took the level of expenditures as given, then the optimal tax rate would be zero. The difference in welfare between these second and third best solutions is of the same order of magnitude as the excess burden of distortionary taxation.

In the US the contemporaneous correlation of detrended GDP with detrended government

consumption is close to zero (see Kydland and Prescott, 1990).² This means that the share of purchases in GDP is highly countercyclical. Suppose the policy maker takes public consumption to be a countercyclical share of GDP. Then optimal taxes would on average be the average share of public consumption in GDP but they would be countercyclical. There would be welfare gains from the average tax and from the countercyclical volatility of the tax.

The Ramsey taxation literature assumes that public spending is exogenous and determines the optimal taxes in response, among other things, to public consumption. A standard question is how taxes should move in response to shocks when public consumption is colinear with output. Whether that colinearity happens because public consumption is an exogenous share of output or because shocks are perfectly correlated makes a significant difference in welfare terms. If the comovement happens because expenditures are indeed a share of output, then the optimal tax solutions in the literature are third best solutions, and there would be substantial welfare gains from taking those comovements into account when choosing taxes.

II. An example economy

Consider a simple economy with a representative household that has preferences over consumption C_t and leisure h_t , described by

$$\sum_{t=0}^{\infty} \beta^t u(C_t, h_t),$$

where

$$u(C_t, h_t) = \ln C_t + \alpha h_t.$$

The technology uses time, $1 - h_t$, to produce a good that can be used for private consumption or public consumption, G_t . The technology is linear and A_t is productivity:

$$C_t + G_t = A_t(1 - h_t) \tag{1}$$

There is exogenous technological progress, so that $A_t = A(1 + \gamma)^t$. The assumptions on public consumption are either that $G_t = G(1 + \gamma)^t$ or that $G_t = gA_t(1 - h_t)$. They are observationally equivalent in terms of growth rates, but have very different implications for optimal policy.

Let

$$G_t = G(1 + \gamma)^t,$$

with G exogenous. The optimal allocation is described by

$$\alpha C_t = A_t,$$

that equates the marginal rate of substitution to the marginal rate of transformation, together with the resource constraint (1). The optimal allocation, defined as the one that maximizes utility subject to the resource constraint, (1) is as follows: Consumption grows at the rate γ and labor is constant,

$$C_t = \frac{A}{\alpha} (1 + \gamma)^t$$

$$h_t = 1 - \frac{1}{\alpha} - \frac{G}{A}.$$

This optimal allocation can be implemented in a competitive equilibrium with lump sum taxes. Agents equate the marginal rate of substitution to the marginal rate of transformation and

lump sum taxes pay for expenditures.

Suppose, now, that

$$G_t = gA_t(1 - h_t)$$

with g exogenous. The resource constraint, (1), is now

$$C_t = (1 - g)A_t(1 - h_t) \tag{2}$$

The optimal allocation, defined again, as the one that maximizes utility subject to the resource constraint, regardless of how it is implemented, is characterized by

$$\alpha C_t = (1 - g) A_t,$$

together with the resource constraint (2). The social marginal rate of transformation is $(1 - g) A_t$, rather than A_t . Labor is less productive because a fraction g of resources is wasted, and therefore consumption is lower. The optimal allocation would then be

$$C_t = \frac{(1 - g) A}{\alpha} (1 + \gamma)^t$$

and

$$h_t = 1 - \frac{1}{\alpha}.$$

In this case there is a wedge between the marginal rate of substitution, αC_t , and the private marginal rate of transformation, $A(1 + \gamma)^t$. The optimal allocations in this case can only be implemented if the government can use proportionate taxes. The government will choose to use

proportionate taxes even if it can tax lump sum.

The two allocations are comparable if we make

$$G_t = gA_t(1 - h_t).$$

For given g , we, then, compare the allocations $C_t = \frac{A}{\alpha}(1 + \gamma)^t$ and $h_t = 1 - \frac{1}{(1-g)\alpha}$ with $C_t = \frac{(1-g)A}{\alpha}(1 + \gamma)^t$ and $h_t = 1 - \frac{1}{\alpha}$. What is the difference in welfare?

The difference in welfare, measured in units of consumption, is given by Δ , such that

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[\frac{A}{\alpha} (1 + \gamma)^t (1 + \Delta) \right] + \alpha \left[1 - \frac{1}{\alpha(1-g)} \right] \right\} \\ = & \sum_{t=0}^{\infty} \beta^t \left\{ \ln \left[\frac{(1-g)A}{\alpha} (1 + \gamma)^t \right] + \alpha \left[1 - \frac{1}{\alpha} \right] \right\} \end{aligned}$$

or

$$\ln(1 + \Delta) = \frac{1}{1-g} - 1 + \ln(1-g)$$

Suppose $g = 0.2$. Then $\ln(1 + \Delta) = .0269$. 2.7% of consumption forever would be the welfare gain of the optimal policy.

Taking expenditures, G_t , as given, when indeed they are a constant share of output, $G_t = gA_t(1 - h_t)$, means that the optimal allocation is a third best, instead of a second best. The difference in welfare is of the same order of magnitude of the excess burden of distortionary taxation. The solution that ignores the response of expenditures to output, is effectively subsidizing consumption at a rate that equals the share of public consumption in output. If that share is 20%, then, the welfare cost is approximately equal to the welfare cost of a tax rate of 20%.

The optimal allocations in the two economies have similar long run and cyclical properties,

but the allocations are very different with significant welfare implications.

Both the long run and the cyclical properties of government purchases in this simple model are different from the ones in the US data. The share of government purchases in GNP in the US has gone up from 10% in 1930 to about 20% in 1985, and it was 19% in 2005. In the post war period, government purchases are acyclical.

If

$$G_t = g_t A_t (1 - h_t),$$

with countercyclical g_t , then the cyclical properties of the two optimal solutions are different. The optimal solution with exogenous g_t has constant labor

$$1 - h_t = \frac{1}{\alpha},$$

and has the movements in g_t reinforce the procyclical behavior of consumption,

$$C_t = \frac{(1 - g_t) A_t}{\alpha} (1 + \gamma)^t.$$

Instead, the optimal solution with exogenous G_t has labor comove with g_t

$$1 - h_t = \frac{1}{(1 - g_t) \alpha}$$

and consumption move with technology only,

$$C_t = \frac{A_t}{\alpha} (1 + \gamma)^t.$$

There are also welfare gains from the alternative cyclical behavior.

III. The economy

The economy is inhabited by a representative household, a representative firm, and a government. Time is discrete and in each period $t = 1, 2, \dots$, one of finitely many events $s_t \in S_t$ occurs. The history of events up to period t , (s_0, s_1, \dots, s_t) , is denoted by $s^t \in S^t$ and the initial realization s_0 is given. Let $\pi(s^t)$ be the probability of occurrence of state s^t .

The technology uses labor $N(s^t)$ to produce private consumption $C(s^t)$ and public consumption $G(s^t)$, according to

$$C(s^t) + G(s^t) = A(s^t)N(s^t)$$

where $A(s^t)$ is productivity. The total use of time for production and leisure $h(s^t)$ is normalized to one.

$$N(s^t) = 1 - h(s^t)$$

The preferences of the households are described by:

$$U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C(s^t), h(s^t), G(s^t)) \quad (3)$$

The sequence of household budget constraints is given by

$$\sum_{s^{t+1}/s^t} q(s^{t+1}/s^t) b^h(s^{t+1}/s^t) \leq b^h(s^t/s^{t-1}) + (1 - \tau^n(s^t)) w(s^t) (1 - h(s^t)) - C(s^t) - T(s^t) \quad (4)$$

where $b^h(s^{t+1}/s^t)$ are state-contingent bonds that pay one unit of consumption in state s^{t+1} and

cost $q(s^{t+1}/s^t)$ in state s^t . $w(s^t)$ is the real wage and $\tau^n(s^t)$ is the labor income tax. $T(s^t)$ are lump sum taxes.

The government finances aggregate expenditures $\{G(s^t)\}_{t=0}^{\infty}$ with lump sum taxes/transfers.

The government can also use labor income taxes.

Households The households maximize utility

$$U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C(s^t), h(s^t), G(s^t)).$$

subject to the sequence of budget constraints, (4), together with the no-Ponzi games condition $\lim_{T \rightarrow \infty} \sum_{s^{T+1}} q(s^{T+1}) b^h(s^{T+1}/s^T) \geq 0$. We assume that $b^h(s^0/s^{-1}) = 0$.

Define

$$q(s^t) = q(s^1/s^0) q(s^2/s^1) \dots q(s^t/s^{t-1}).$$

We also define the return on a noncontingent bond paying $R(s^t)$ in $t+1$, for each unit of consumption in t , as

$$R(s^t) = \frac{1}{\sum_{s^{t+1}/s^t} q(s^{t+1}/s^t)}.$$

The sequence of budget constraints can be written as the single budget constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) \{C(s^t) - (1 - \tau^n(s^t)) w(s^t) (1 - h(s^t)) + T(s^t)\} \leq 0$$

The first order conditions of the households problem include

$$\frac{u_C(s^t)}{u_h(s^t)} = \frac{1}{(1 - \tau^n(s^t)) w(s^t)}$$

$$\frac{u_C(s^t)}{\beta \pi(s^{t+1}/s^t) u_C(s^{t+1})} = \frac{q(s^t)}{q(s^{t+1})}$$

$$q(s^{t+1}) = q(s^t) q(s^{t+1}/s^t)$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) \{C(s^t) - (1 - \tau^n(s^t)) w(s^t) (1 - h(s^t)) + T(s^t)\} = 0$$

Firms Firms are competitive. They maximize profits

$$A(s^t) N(s^t) - w(s^t) N(s^t).$$

This implies

$$w(s^t) = A(s^t).$$

Government The government in this economy either takes aggregate public consumption, $G(s^t)$, as exogenous, or, alternatively, takes the share of expenditures on output as exogenous.

The policy variables are public consumption, $G(s^t)$, lump sum taxes, $T(s^t)$, taxes on labor income, $\tau^n(s^t)$, and debt supplies, $b^g(s^t)$ for all $t \geq 0$ and states $s^t \in S^t$.

If the budget constraint of the households and the market clearing conditions hold, the budget constraint of the government also holds.

Equilibria An equilibrium in this economy can be defined as an allocation $\{C(s^t), h(s^t), N(s^t)\}$, prices $\{q(s^{t+1}/s^t), q(s^t), R(s^t), w(s^t)\}$, and policies $\{\tau^n(s^t), \tau^c(s^t), T(s^t)\}$ that solve the problems of the households, the firms, and the government, and such that markets clear.

An equilibrium is characterized by the following conditions:

$$\frac{u_C(s^t)}{u_h(s^t)} = \frac{1}{(1 - \tau^n(s^t)) w(s^t)}$$

$$\frac{u_C(s^0)}{\beta^t \pi(s^t) u_C(s^t)} = \frac{1}{q(s^t)}$$

$$u_C(s^t) = R(s^t) \sum_{s^{t+1}/s^t} \beta \pi(s^{t+1}/s^t) u_C(s^{t+1})$$

$$q(s^{t+1}) = q(s^t) q(s^{t+1}/s^t)$$

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t) \{C(s^t) - (1 - \tau^n(s^t)) w(s^t) (1 - h(s^t)) + T(s^t)\} = 0$$

$$w(s^t) = A(s^t)$$

$$C(s^t) + G(s^t) = A(s^t) N(s^t)$$

$$N(s^t) = 1 - h(s^t)$$

The implementability conditions on the allocations $\{C(s^t), h(s^t)\}$ are the resource constraints

$$C(s^t) + G(s^t) = A(s^t) (1 - h(s^t)), \quad (5)$$

only. The other constraints determine other variables:

$$w(s^t) = A(s^t)$$

determines $w(s^t)$;

$$\frac{u_C(s^t)}{u_h(s^t)} = \frac{1}{(1 - \tau^n(s^t)) w(s^t)}$$

determines $\tau^n (s^t)$;

$$\frac{u_C (s^0)}{\beta^t \pi (s^t) u_C (s^t)} = \frac{1}{q (s^t)}$$

determines $q (s^t)$;

$$u_C (s^t) = R (s^t) \sum_{s^{t+1}/s^t} \beta \pi (s^{t+1}/s^t) u_C (s^{t+1})$$

determines $R (s^t)$;

$$q (s^{t+1}) = q (s^t) q (s^{t+1}/s^t)$$

determines $q (s^{t+1}/s^t)$;

$$\sum_{t=0}^{\infty} \sum_{s^t} q (s^t) \{C (s^t) + T (s^t) - (1 - \tau^n (s^t)) w (s^t) (1 - h (s^t))\} = 0$$

restricts $T (s^t)$;

$$N (s^t) = 1 - h (s^t)$$

determines $h (s^t)$.

IV. Second best policy: Optimal tax policy for exogenous public spending

Exogenous level of public consumption If $G (s^t)$ is exogenous, the optimal allocations maximize utility (3) subject to the single implementability condition (5), the resource constraint.

The optimal allocations are characterized by

$$\frac{u_C (s^t)}{u_h (s^t)} = \frac{1}{A (s^t)}$$

and

$$C(s^t) + G(s^t) = A(s^t) [1 - h(s^t)]. \quad (6)$$

The implementation is done with lump sum taxes only, that pay for government expenditures. The labor income tax is not used,

$$\tau^n(s^t) = 0.$$

Even if there are lump sum taxes, the optimal allocation is a second best allocation because there are restrictions on $G(s^t)$.

If public spending was endogenous, the first best conditions would be

$$\frac{u_G(s^t)}{u_C(s^t)} = 1$$

and

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t)$$

as well as the resource conditions (6). The restrictions on $G(s^t)$ distort the margin between private and public consumption, but not between consumption and leisure.

Exogenous share of public consumption If, instead, $g(s^t)$ is exogenous, the optimal allocation is the solution of the following maximization problem:

$$\text{Max } U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C(s^t), h(s^t), g(s^t) A(s^t) [1 - h(s^t)]) \quad (7)$$

s.t.

$$C(s^t) \leq [1 - g(s^t)] A(s^t) [1 - h(s^t)]$$

The solution is described by

$$\beta^t \pi(s^t) u_C(s^t) - \lambda(s^t) = 0,$$

$$\beta^t \pi(s^t) u_h(s^t) - \beta^t \pi(s^t) u_G(s^t) g(s^t) A(s^t) - \lambda(s^t) [1 - g(s^t)] A(s^t) = 0,$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)].$$

The optimal allocations are, then, such that

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t) \left\{ 1 - \left(1 - \frac{u_G(s^t)}{u_C(s^t)} \right) g(s^t) \right\}.$$

For the case in which $u_G(s^t) = 0$, and therefore G does not appear in the utility, the optimal margin between consumption and leisure is

$$\frac{u_h(s^t)}{u_C(s^t)} = [1 - g(s^t)] A(s^t).$$

Then the second best policy is to set

$$\tau(s^t) = g(s^t).$$

If expenditures were endogenous, then

$$\frac{u_G(s^t)}{u_C(s^t)} = 1$$

and

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t).$$

This solution is the first best solution, in which public expenditures are endogenous. The exogeneity of public spending creates distortions in the margin between private and public consumption, but it also creates a distortion in the margin between consumption and leisure.

If $\frac{u_G(s^t)}{u_C(s^t)} < 1$, there is a distortion. If $\frac{u_h(s^t)}{u_C(s^t)} = A(s^t)$ consumption would be too high relative to leisure. It is optimal to set a wedge in the consumption/leisure margin. The optimal wedge is $\left\{ 1 - \left(1 - \frac{u_G(s^t)}{u_C(s^t)} \right) g(s^t) \right\}$. The wedge is the highest for a given $g(s^t)$ when $u_G(s^t) = 0$, and the lowest when $\frac{u_G(s^t)}{u_C(s^t)} \rightarrow 1$. In as much as G is useless, and G is a share of output, it is optimal to take that into account and reduce the level of output.

In summary, taking expenditures to be exogenous means that, in general, the first best cannot be attained. The optimal solution is second best. Adding a distortion can then be optimal. This is a simple application of the second best principle.

When $g(s^t)$ is exogenous and there is a distortion in the margin between private and public consumption, $\frac{u_G(s^t)}{u_C(s^t)} < 1$, then it is optimal to also impose a wedge in the margin between consumption and leisure. In this case, the policy that eliminates the distortion in the consumption-leisure margin,

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t),$$

which would be optimal if $G(s^t)$ was exogenous, is third best.

Welfare Suppose public spending is an exogenous share of output. The welfare difference between the second-best solution that takes that into account and the third best solution that does

not distort between consumption and leisure is of first order.

Suppose, for now that $u_G = 0$. How does the solution characterized by

$$\frac{u_h(s^t)}{u_C(s^t)} = [1 - g(s^t)] A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)],$$

compare to the solution characterized by

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)]?$$

To simplify the calculations, let the utility function be

$$U = \ln C(s^t) + \alpha h(s^t)$$

Then the marginal conditions

$$\frac{u_h(s^t)}{u_C(s^t)} = [1 - g(s^t)] A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)],$$

become

$$\alpha C(s^t) = [1 - g(s^t)] A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)],$$

or

$$h(s^t) = 1 - \frac{1}{\alpha}$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) \frac{1}{\alpha}.$$

In the other case, without the distortion in the consumption-leisure margin, the conditions are

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)],$$

or

$$\alpha C(s^t) = A(s^t)$$

and

$$C(s^t) = [1 - g(s^t)] A(s^t) [1 - h(s^t)].$$

Thus,

$$h(s^t) = 1 - \frac{1}{\alpha [1 - g(s^t)]}$$

and

$$C(s^t) = \frac{A(s^t)}{\alpha}.$$

In order to compute the welfare differences, we have

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u\left(\frac{A(s^t)}{\alpha} (1 + \Delta), 1 - \frac{1}{\alpha [1 - g(s^t)]}\right) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u\left(\frac{[1 - g(s^t)] A(s^t)}{\alpha}, 1 - \frac{1}{\alpha}\right) \quad (8)$$

or

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left\{ \ln(1 + \Delta) - \frac{1}{1 - g(s^t)} - \ln \frac{1 - g(s^t)}{\alpha} + 1 \right\} = 0 \quad (9)$$

The welfare loss is independent of $A(s^t)$. Suppose $g(s^t) = g$. Then,

$$\ln(1 + \Delta) = \frac{1}{1 - g} + \ln \frac{1 - g}{\alpha} - 1. \quad (10)$$

Suppose $g = .2$, $\alpha = 1$. Then the welfare loss is huge, around 2.7% of consumption forever.

V. Transfers

Does it make a difference whether spending is public consumption or transfers? If public spending is transfers back to the households, the results on the optimal taxes are unaffected by the exogeneity assumptions, provided transfers do not enter the utility function, or the marginal utility is equal to zero, as is the optimal solution for transfers.

To see this, consider the problem where utility depends on transfers and transfers are an exogenous share of production. The assumption that preferences depend on transfers is questionable, since they would not be the aggregate preferences of individual agents under the conditions

of Gorman aggregation.

The optimal allocation is the solution of the following problem:

$$\text{Max } U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C(s^t), h(s^t), Tr(s^t)) \quad (11)$$

subject to

$$C(s^t) \leq A(s^t) [1 - h(s^t)]$$

where

$$Tr(s^t) = tr(s^t) A(s^t) [1 - h(s^t)]$$

The solution is described by

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t) \left\{ 1 + \frac{u_{Tr}(s^t)}{u_C(s^t)} tr(s^t) \right\}$$

and

$$C(s^t) = A(s^t) [1 - h(s^t)].$$

For the case in which $u_{Tr}(s^t) = 0$, which is the case when transfers are not in the utility function, there is no wedge between consumption and leisure,

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t),$$

just as in the case where the level of transfers is exogenous. The second best policy is to set the labor income tax equal to zero, and transfers are solely financed with lump sum taxes.

If transfers were endogenous, then the optimal choice of transfers would also be

$$\frac{u_{Tr}(s^t)}{u_C(s^t)} = 0,$$

so that

$$\frac{u_h(s^t)}{u_C(s^t)} = A(s^t).$$

For any case where transfers increase the utility of the representative agent and they are not optimally chosen, then the exogeneity of transfers affects the optimal margin between consumption and leisure. If $\frac{u_{Tr}(s^t)}{u_C(s^t)} > 0$, it is optimal to impose a wedge between consumption and leisure. For the case where transfers are low enough that $\frac{u_{Tr}(s^t)}{u_C(s^t)} = 1$, the optimal wedge would be the share of transfers in utility,

$$\frac{u_h(s^t)}{u_C(s^t)} = [1 + tr(s^t)] A(s^t)$$

so that it would be optimal to set the labor income tax equal to the share of transfers.

Again, in this case, the assumption of exogeneity would matter for the optimal taxation policy. If the share of transfers in output is exogenous, and transfers are below the optimum, then it is optimal to distort the margin between consumption and leisure, which is, again, an application of the second best principle.

VI. Third best results, when there are no lump sum taxes

We now assume that lump-sum taxes cannot be used, and ask the same question as before. How do the two solutions differ depending on the assumptions on public spending? The Ramsey analysis usually assumes that $G(s^t)$ is exogenous and computes the optimal solution. It is common to look at the optimal solution when $G(s^t)$ is a constant fraction of output. How important is it that this

relationship is not taken into account?

When $G(s^t) = g(s^t) A(s^t) N(s^t)$, the Ramsey problem is

$$\text{Max } U = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C(s^t), h(s^t)) \quad (12)$$

subject to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \{u_C(s^t) C(s^t) - u_h(s^t) (1 - h(s^t))\} = 0$$

$$C(s^t) \leq [1 - g(s^t)] A(s^t) [1 - h(s^t)]$$

For the case of separable utility with constant intertemporal elasticity of substitution, $\frac{1}{\sigma}$, in consumption and linear leisure, the first order conditions are:

$$\beta^t \pi(s^t) \{u_C(s^t) + \Psi [u_{CC}(s^t) C(s^t) + u_C(s^t)]\} - \lambda(s^t) = 0$$

or

$$\beta^t \pi(s^t) \{u_h(s^t) + \Psi u_{hh}(s^t)\} - \lambda(s^t) [1 - g(s^t)] A(s^t) = 0$$

or

$$\beta^t \pi(s^t) u_C(s^t) [1 + \Psi (1 - \sigma)] = \lambda(s^t)$$

$$\beta^t \pi(s^t) u_h(s^t) [1 + \Psi] = \lambda(s^t) [1 - g(s^t)] A(s^t)$$

It follows that

$$\frac{u_C(s^t)}{u_h(s^t)} = \frac{1 + \Psi}{[1 + \Psi (1 - \sigma)] [1 - g(s^t)] A(s^t)}$$

The optimal solution is implemented with a labor income tax that is proportional to the share of

government spending. The tax is constant when $g(s^t) = g$ is constant.

If the planner does not take into account the fact that expenditures depend on output, the solution is

$$\frac{u_C(s^t)}{u_h(s^t)} = \frac{1 + \Psi}{[1 + \Psi(1 - \sigma)] A(s^t)}$$

In this case, the proportionate distortion is constant always. The solution can be implemented with a constant labor income tax. To implement this solution, when indeed $G(s^t) = g(s^t) A(s^t) N(s^t)$, is fourth best.

The two solutions are the same when the share of government spending in output is constant, $g(s^t) = g$. In this case there is no level effect. Because lump-sum taxes cannot be used, the average level of the tax is given by the share of spending in output, in both cases. For this particular structure, with these particular preferences, the difference between the two solutions is in the comovement of taxes with the share of public spending in output. If it is the case that public spending is an exogenous fraction of output, then taxes should move with that fraction, in order to achieve a third best solution, rather than a fourth best, where those taxes would be constant.

VII. Concluding remarks

The analysis of optimal taxation in dynamic general equilibrium models usually assumes that the level of public spending is exogenous. In an environment with lump sum taxes, this second-best assumption is relevant for the margin between private and public consumption, but does not affect the optimal wedges for private decisions. In particular, the optimal wedge in the consumption-leisure margin is not affected by the assumption of exogeneity. This means that whether the planner can decide on public spending or not, the, respectively, first and second best solutions will be implemented by the same taxes. In particular, it is always optimal to use lump-sum taxes, and

to use them only. Labor income or consumption taxes should not be used.

Instead, if it is the share of public consumption in output that is exogenous, then the assumption of exogeneity is no longer innocuous. While if public expenditures are decided jointly with taxes, the first best solution can be achieved, instead, if public spending is exogenous, then the second best solution imposes optimal wedges in the private decisions. In particular, even if lump sum taxes can be used, it may be optimal to tax labor income, or consumption, at a rate approximately equal to the share of public consumption in output.

This means that, under the assumption of exogenous public spending, whether it is the level or the share of public spending that is exogenous, optimal policy can give welfare differences of the order of magnitude of the excess burden of taxation. When the level of public spending is exogenous, distortionary taxation is a burden, that can be 3% of consumption for ever, or even higher. Instead, when the share of spending is exogenous, it is lump sum taxation that is a burden of that same order of magnitude.

In the Ramsey benchmark in which lump-sum taxes cannot be used, the implications of the exogeneity assumption are similar, even if the welfare implications are less relevant. If the level of public consumption is exogenous, the assumption of exogeneity affects the level but not the cyclical properties of the tax rates. Instead, if it is the share of public consumption in output that is exogenous, then the exogeneity assumption has implications both on the level and on the cyclical properties.

These conclusions question the usefulness of the separability assumptions in the optimal choices of expenditures and taxes, which are the standard in optimal policy analysis.

REFERENCES

- [1] Chari, V. V. and Patrick Kehoe. 1999. "Optimal Fiscal and Monetary Policy." In *Handbook of Macroeconomics*, vol 1C, edited by John Taylor and Michael Woodford. North-Holland.
- [2] Kydland, Finn E. and Edward C. Prescott. 1990. "Business Cycles: Real Facts and a Monetary Myth." *Quarterly Review, Federal Reserve Bank of Minneapolis*, Spring: 3-18.
- [3] Lucas, Robert E., Jr. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy without Capital." *J. Monetary Econ.* 12, 1: 55-93.
- [4] Mirrlees, James A. 1971. "An Exploration in the Theory of Income Taxation." *Rev. of Econ. Stud.* 38 (April): 75-208.
- [5] Ramsey, Frank P.. 1927. "A Contribution to the Theory of Taxation." *Econ. J.* 37, 145: 47-61.

NOTES

I thank Isabel Correia for very useful discussions. Financial support of FCT is gratefully acknowledged.

¹For Ramsey taxation in dynamic general equilibrium, see Lucas and Stokey (1983) or Chari, Christiano and Kehoe (1999)

²In Portugal the same correlation is 0.36 (Correia, Neves and Rebelo, 1995).