Is the Friedman rule optimal when money is an intermediate good?

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Abstract

In contrast to the recent literature on the optimal inflation tax, we show that, in models where money reduces transactions costs, it is optimal to set the inflation tax to zero when seigniorage is replaced by revenue from distortionary taxes. The main reasons for this result are that the variable costs of supplying real balances are negligible and the inflation tax is a unit tax. We also show that the intermediate good optimal taxation rules in the public finance literature cannot be directly applied, both when money is costless and when it requires resources to be produced.

Key words: Inflation tax; Transactions technology; Intermediate good; Friedman rule

JEL classification: E31; E41; E58; E62

1. Introduction

Friedman (1969) proposed a monetary policy rule that may generate nominal interest rates, on assets with a riskless nominal return, as low as possible. This means setting the revenue from seigniorage equal to zero. In his own words: the 'rule for the optimum quantity of money is that it will be attained by a
rate of price deflation that makes the nominal rate of interest equal to zero'. The arguments invoked by Friedman were simple first best, partial equilibrium arguments. A good that is costless to produce should be priced at zero, is a simple version of that argument. The main criticism to the Friedman rule is attributed to Phelps (1973) that applied standard Ramsey (1927) taxation principles: In the absence of lump-sum taxes, the optimum taxation problem is one of financing a given level of government expenditures in the least distorting manner. In this context, the marginal distortion caused by one unit of revenue collected with one tax should be equalized across the different taxes. The standard implication was that an optimal inflation tax policy would result in a strictly positive nominal interest rate.

Recent developments in dynamic, general equilibrium monetary theory have however generated second-best taxation rules that are compatible with the Friedman rule. The purpose of this paper is to analyze the optimality of the Friedman rule in monetary economies where transactions require real balances and time. The fiscal environment is such that exogenous government expenditures must be financed with either the inflation tax or a distortionary income tax. Inflation introduces a distortion in the use of time for transactions.

The economic environment in this paper is similar to the ones in Kimbrough (1986a), Faig (1986, 1988), Guidotti and Végh (1993), Chari, Christiano, and Kehoe (1993). All these authors agree that the Friedman rule is optimal when the transactions costs function (defined as time spent in transactions as a function of the level of transactions and real balances) is homogeneous of degree one. But, while Guidotti and Végh (1993) claim that this is a necessary condition for an optimal zero nominal interest rate, Faig (1986) and Chari, Christiano, and Kehoe (1993) show that the optimality of the Friedman rule extends to transactions costs functions of degree of homogeneity greater than one and Kimbrough (1986b) conjectures, with no explicit proof, that real balances should not be taxed in 'any economy in which, in equilibrium, scarce resources are used up in the transaction process and agents can economize on these transactions costs by holding money'. These conflicting results justify the present work. Also, the absence of a general result for functions of degree of homogeneity less than one is particularly unsatisfactory since the transactions functions supported by the rationale provided by Baumol (1952) and Tobin (1956) are homogeneous of degree zero (see also Lucas, 1994).

In this paper we show that in the setup of Kimbrough (1986a), Faig (1986, 1988), Guidotti and Végh (1993), and Chari, Christiano, and Kehoe (1993), where the costs of producing real money are assumed to be negligible, the Friedman rule is the optimal policy for any homogeneous transactions costs functions. It is therefore a general result. However, if the costs of producing money were not negligible and were a function of the real quantity of money, then the optimal taxation of real balances would be directly related to the degree of homogeneity of the transactions costs function. In any case, the intermediate good
optimal taxation rules developed by Diamond and Mirrlees (1971) could not be directly applied there. We provide the intuition for these results by building up fictitious real economies. We derive the ad-valorem tax rules on intermediate goods for homogeneous production functions in the simplest possible framework and in a production and tax structure corresponding to the monetary economy. The optimal rules in this last environment are not a direct application of the results in Diamond and Mirrlees (1971) due to the particular production structure and the restrictions on taxation resulting from transactions being produced in the household. Furthermore we compute the optimal unit taxes when the costs of producing the intermediate good become arbitrarily small. The optimal unit taxes are equal to zero and the intuition is that the unit price of a good that has an arbitrarily small cost of production should also be very small. Otherwise the ad-valorem tax would have to be arbitrarily large. Since real balances are taken to be a free good and the nominal interest rate is by construction a unit tax, the optimal nominal interest rate is zero, whatever is the degree of homogeneity of the transactions costs function.

The relevance of the work on the optimum quantity of money, and particularly of the policy rule that governments should not resort to the inflation tax, would be questionable if the welfare gains from reducing inflation to the Friedman rule, while positive, were insignificant. Early work by Bailey (1956), and others, seemed to point in this direction. Lucas (1994) develops a transactions technology model, calibrated to US secular data on the money demand schedule, and calculates the gains from reducing the inflation rate at the US present day levels to the first best rule of a zero nominal interest rate. The annual gains are roughly 1% of GNP. As he points out, ‘this is real money’.

The paper proceeds as follows. In Section 2 we present the setup, define the private problem and the Ramsey solution, and determine the optimal inflation tax. In Section 3 we derive the optimal solution with variable costs of producing money. In Section 4 we provide the intuition for the results obtained in Sections 2 and 3 and relate them to the intermediate goods taxation rules. Section 5 contains the conclusions.

2. The optimal inflation tax rule

2.1. Setup

The model is similar to the one described in Kimbrough (1986a), Faig (1988), Guidotti and Végh (1993), and Chari, Christiano, and Kehoe (1993). There is a large number of identical households that are endowed with one unit of time that can be used as leisure, in transactions, or allocated to the production of one good. In Section 3, we also consider the possibility that the production of money requires time. The technology to produce the good is linear with a
unitary coefficient. Transactions are costly. This cost is measured in terms of time allocated to the activity. Money can reduce this cost. The households have preferences defined over a consumption good and leisure.

In each period there are markets for goods and labor and markets for assets. There are two assets, money and nominal bonds. There is a benevolent government that must finance a given constant level of government expenditures with income taxes or with an inflation tax.

2.2. The private problem

The households seek to maximize

$$\sum_{t=0}^{\infty} \beta^t U(c_t, h_t),$$

where $U$ is an increasing concave function, $c_t$ are consumption goods, and $h_t$ is leisure at time $t$. The households supply labor $1 - h_t - s_t$, where $s_t$ is time spent in transactions. The households choose holdings of money, $M_t$, and nominal bond holdings, $B_t$. These bonds entitle the households to $(1 + i_t)B_t$ units of money in period $t + 1$. The restrictions of the private problem are the budget constraints, for $t \geq 0$, with initial conditions, $M_{-1} = B_{-1} = 0$, and together with a no-Ponzi games condition,

$$p_t c_t + M_t + B_t \leq M_{t-1} + (1 + i_{t-1})B_{t-1} + p_t(1 - \tau_t)(1 - h_t - s_t),$$

and the transactions technology,

$$s_t \geq l(c_t, M_t/p_t),$$

$\tau_t$ is the income tax rate.

Let $m_t = M_t/p_t$. We assume that the function $l$ is homogeneous of degree $k$ and so it can be written as $l(c, m) = L(m/c)c^k$. $L$ is characterized by $L' \leq 0$ and $L'' \geq 0$, so that an increase in the real quantity of money decreases the time spent with transactions at a decreasing rate. The point of 'satiation' in real balances, $m/c$, is defined as $L(m/c) = l_m(m/c) = 0$. It is not worthwhile to increase $m$ beyond this point since by doing it, it is not possible to save resources.

When $k = 0$ and $L(m/c) = \eta(c/m)$, the form for the transactions technology can be justified by assuming, inspired in Baumol (1952) and Tobin (1956), that the consumer spends cash holdings intended for the purchase of the good at a constant rate $c_t$ per unit of time. $c_t/m_t$ is the number of times cash balances for

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1 For convenience we impose as initial conditions zero holdings of nominal assets. The results on the optimality of the Friedman rule are not affected by this assumption.

2 The restriction on the second derivative of the transactions function assures that the isoquants of the production function of transactions are convex and that the demand for money depends negatively on the nominal interest rate.
transactions of the good are exhausted and must be restored, the number of trips to the bank. This time cost per trip to the bank is a constant \( \eta \).

Optimality of the private sector choices requires that the marginal benefit of using one unit of real money in terms of time be equal to its opportunity cost, the inflation tax, \( I \), described by the following first-order condition, for \( t \geq 0 \).

\[
- l_m(t) = \frac{1}{1 - \tau_t} I_t,
\]

(2.4)

where \( I_t = \frac{i_t}{1 + \tau_t}, 0 \leq I_t \leq 1 \).

The optimal choice of real balances, \( m \), in (2.4), is such that the private agents choose the point where the private value of using money, \( -l_m(1 - \tau) \), is equal to its opportunity cost, \( I \). The implementation of the Friedman rule, \( I = 0 \), implies that \( l_m = 0 \), which means that the agents choose the point of satiation in real balances and no more resources can be saved by increasing the amount of real money per unit of transactions.

2.3. The Ramsey problem

The solution of the Ramsey problem\(^3\) is an allocation and a set of prices and policy variables such that welfare is maximized and the allocation can be decentralized as a competitive equilibrium. The restriction that guarantees that the allocation can be decentralized is an implementability condition, that is constructed by replacing the taxes and prices by the expressions for quantities, from the first-order conditions of the private problem. Imposing that the transactions function is homogeneous of degree \( k \), the implementability condition is the following expression:

\[
\sum_{t=0}^{\infty} \beta^t [U_c(t)c_t - U_h(t)(1 - h_t) + U_k(t)(1 - k)l(t)] = 0.
\]

(2.5)

So the Ramsey problem is defined as: Choose \( \{c_t, h_t, m_t\}_{t=0}^{\infty} \) to maximize (2.1) subject to the implementability condition, (2.5), and the resources constraints, for \( t \geq 0 \),

\[
c_t + g_t \leq 1 - h_t - l(t),
\]

(2.6)

where \( g_t \) are government expenditures at time \( t \).

The marginal condition for the optimal choice of real balances in this second best problem is the following:

\[
[\beta^t \psi U_h(t)(1 - k) - \lambda_t] l_m(t) = 0.
\]

(2.7)

The multipliers of the implementability condition and the resources constraints, respectively \( \psi \) and \( \lambda_t \), \( t \geq 0 \), are strictly positive at the optimum, since they

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\(^3\) See the Appendix for the full characterization of the Ramsey solution.
measure, respectively, the excess burden of taxation and the shadow price of resources in the economy. The following proposition states the optimality of the Friedman rule:

**Proposition 1.** The Ramsey solution is such that $I_t = 0$, i.e., the Friedman rule is optimal, for any homogeneous transactions costs function.\(^4\)

**Proof** (sketch). Eq. (2.7) is satisfied when $l_m(t) = 0$ or $\beta^t \psi U_h(t)(1 - k) - \lambda_t = 0$. We show that $\beta^t \psi U_h(t)(1 - k) - \lambda_t = 0$ is not a solution of the problem. Notice that when $k \geq 1$, this expression alone implies that the multipliers will either be zero or have opposite signs. That solution is excluded since the two multipliers must be strictly positive. To show that, in general (for $k < 1$), that is not the solution requires using the other first-order conditions of the Ramsey problem. We obtain a contradiction in the signs of the multipliers. The multiplier of the implementability condition, $\psi_t$, would be negative, and when $k < 1$, so is the shadow price of the resources constraint. So $\beta^t \psi U_h(t)(1 - k) - \lambda_t = 0$ cannot characterize the maximum of the Ramsey problem. $l_m(t) = 0$ is the only solution that satisfies the first-order conditions with positive multipliers. This solution can be decentralized by $I_t = 0$ (from (2.4)). \(\square\)

### 3. Money as an intermediate good

The purpose of this and the following sections is to interpret the general result of optimality of the Friedman rule derived in the previous section. Optimality results in the class of general equilibrium monetary models studied in this paper are usually interpreted as relating to the optimal taxation rules of intermediate goods in the literature of public finance, specifically the ones developed by Diamond and Mirrlees (1971). This intermediate goods interpretation of the optimality of the Friedman rule in transactions technology monetary models goes back to Kimbrough (1986a). Guidotti and Végh (1993) have also invoked the principles of optimal taxation of intermediate goods to justify their claim that the Friedman rule is optimal only when the transactions costs function is constant returns to scale. On the other hand, Chari, Christiano, and Kehoe (1993) claim that their results are not an extension of Diamond and Mirrlees (1971) result. In this framework the qualification of money as an intermediate good serves the purpose of distinguishing it from a final good. However, the fact that money is typically assumed to be costless to produce would more precisely qualify it as a

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\(^4\)The Ramsey solution of this problem is time-consistent. This is so because the initial total nominal assets are zero and the problem is stationary.

\(^5\)See the Appendix for the complete proof.
free primary input rather than an intermediate good. It turns out, as we show in this section, that this distinction does matter.

In this section we derive the optimal inflation tax results when money requires resources for its production, so that it is literally an intermediate good rather than a free primary input. As in Chari, Christiano, and Kehoe (1993), we assume that one unit of real balances requires $1/\alpha$ units of time. We allow for differing tax rates on labor used in the production of the consumption good, and the production of money. $n_{1t}$, labor used in the production of the consumption good, is taxed at rate $\tau_{1t}$, and $n_{2t}$, labor used in the production of money, is taxed at rate $\tau_{2t}$. The only possibility of paying for the use of money is the opportunity cost of money, $I_t$. The wage rate before taxes, expressed in terms of the consumption good, in the consumption good sector is equal to one, since the technology is linear with a unitary coefficient. The wage rate in the money-producing sector is 

$$w_2t = \frac{(1 - \tau_{1t})}{(1 - \tau_{2t})}$$

since the after-tax wages must be equalized across sectors. The budget constraints for the households are defined, for $t \geq 0$, by

\begin{align*}
  p_t c_t + M_t + B_t &\leq M_{t-1} + (1 + i_{t-1})B_{t-1} + p_t (1 - \tau_{1t}) n_{1t} + p_t (1 - \tau_{2t}) w_2t n_{2t}, \\
  h_t + s_t + n_{1t} + n_{2t} &\leq 1.
\end{align*}

(3.1)

(3.2)

The transactions technology is given by (2.3) and the technology to produce real balances is described by

$$M_t / p_t = \alpha n_{2t}.$$ 

The resources constraint is

$$c_t + g_t \leq 1 - h_t - I(c_t, m_t) - n_{2t}.$$ 

Since $1 - \tau_{1t} = (1 - \tau_{2t})w_2t$ the budget constraint can be written as

$$p_t c_t + M_t + B_t \leq M_{t-1} + (1 + i_{t-1})B_{t-1} + p_t (1 - \tau_{1t})(1 - h_t - s_t).$$

(3.3)

which is the same condition as (2.2) once $\tau_t$ is replaced by $\tau_{1t}$. The restrictions of the private problem are the budget constraints (3.3) and the transactions technology (2.3), for $t \geq 0$. The first-order conditions of the private problem are identical to the ones in the problem in Section 2, but with $\tau_t$ replaced by $\tau_{1t}$. We thus have

$$- l_m(t) = \frac{1}{1 - \tau_{1t}} I_t.$$ 

(3.4)

The expression for the implementability condition is the same as (2.5) and the Ramsey problem is to choose \{c_t, h_t, m_t\}_{t=0}^{\infty} to maximize welfare, (2.1), subject to the implementability condition, (2.5), and the resources constraints, for $t \geq 0$,

$$c_t + g_t \leq 1 - h_t - l(c_t, m_t) - m_t/\alpha.$$ 

(3.5)
An interior solution of the Ramsey problem requires the following condition, for \( t \geq 0 \),

\[
\beta' \psi U_h(t)(1 - k)l_m(t) - \lambda_t \left[ l_m(t) + 1/\alpha \right] = 0,
\]
where \( \psi \) and \( \lambda_t, t \geq 0 \), are the multipliers associated with the implementability condition, (2.5), and the resources constraints, (3.5), respectively. Condition (3.6) differs from condition (2.7), for the problem without costs of producing money, in the extra term \( 1/\alpha \).

From (3.6), the solution is

\[
l_m(t) = \frac{\lambda_t}{\beta' \psi U_h(t)(1 - k) - \lambda_t \left( \frac{1}{\alpha} \right)}. \tag{3.7}
\]

Using the necessary conditions of the private problem (3.4), we obtain the following optimal taxation rules:

\[
\begin{align*}
\frac{1}{1 - \tau_{1t}} I_t &= \frac{1}{\alpha}, & \text{when } & k = 1, \\
\frac{1}{1 - \tau_{1t}} I_t &> \frac{1}{\alpha}, & \text{when } & k < 1, \tag{3.8} \\
0 &\leq \frac{1}{1 - \tau_{1t}} I_t < \frac{1}{\alpha}, & \text{when } & k > 1.
\end{align*}
\]

In this framework with costs of producing money, the nominal interest rate cannot be identified with the tax on real balances. The government pays for real balances at the shadow price of this good, \( p_m \), and consequently the nominal interest rate can be decomposed in two additive terms: the price of real balances and the unit tax on money. Alternatively we can consider that the tax on real balances is ad-valorem. A zero tax on real balances implies a positive nominal interest rate, covering the price of producing the real balances. For the same reason it is possible in this case to have a negative tax on real balances, since it can be decentralized through a positive nominal interest rate. Let us then, decompose the nominal interest rate into two parts, by defining the ad-valorem tax on real money, \( \tau_{mt} \), as

\[
\tau_{mt} = I_t / p_{mt} - 1, \tag{3.9}
\]

where \( p_{mt} \) is the production price of real balances in units of time. In equilibrium, it is given by

\[
p_{mt} = \frac{w_{2t}}{\alpha} = \frac{1 - \tau_{1t}}{1 - \tau_{2t}} \frac{1}{\alpha}. \tag{3.10}
\]

\[\text{Notice that, as we saw in the previous section, } \beta' \psi U_h(t)(1 - k) - \lambda_t \neq 0 \text{ must hold, for the multipliers to be positive.}\]
Because $I$ is decomposed in the tax $\tau_m$ and in the price of real balances, and because the price of real balances, for a given $\tau_1$, depends on $\tau_2$, it is clear that the optimal level of $I$, derived from the Ramsey solution does not allow for the independent determination of both the optimal level of the ad-valorem tax on real balances, $\tau_m$, and the tax on labor income generated in the production of real balances, $\tau_2$. The government has these two sources of revenue, resulting from the use of money: The tax on holding money, defined as the proportional difference between the nominal interest rate and the price of real balances, and the tax on labor used in the production of real balances. These two sources of tax revenues can be thought of as just one joint ad-valorem tax on real money, $\Theta_{mt}$, defined as

$$\Theta_{mt} \equiv \tau_{mt} + \tau_{2t}. \quad (3.11)$$

There is a correspondence between this tax and the optimal taxation rules in (3.8), that is summarized in the following proposition:

**Proposition 2.** The Ramsey solution is characterized by a zero joint tax on real balances, $\Theta_{mt} = 0$, if the transactions function $l$ is homogeneous of degree one. If the function is homogeneous of degree $k < 1$, then $\Theta_{mt} > 0$; if it is homogeneous of degree $k > 1$, then $\Theta_{mt} < 0$.

**Proof.** Using the definitions of $\Theta_{mt}$, (3.11), and $p_{mt}$, (3.10), we obtain the following expression:

$$\Theta_{mt} \equiv (1 - \tau_{2t})\alpha [I_t/(1 - \tau_{1t}) - 1/\alpha].$$

The results are straightforward once the optimal taxation rules in (3.8) are applied.

From Proposition 2, when the transactions function is constant returns to scale, the joint tax on real money is zero, $\Theta_m = 0$. When the tax on labor income in the money producing sector is zero ($\tau_2 = 0$), the tax on real balances is also zero, $\tau_m = 0$. But if it is not possible to have distinct tax rates on labor in the two taxable sectors, the optimal taxation rules imply that it is optimal to subsidize money, so $\tau_m < 0$. So, although labor is taxed at the same rate in the two sectors, the intermediate good should be subsidized. Therefore it is clear that Proposition 2 is not a simple extension of Diamond and Mirrless (1971) results.

When the degree of homogeneity of the transactions costs function is different from one, then the optimal ad-valorem joint tax on real money is different from zero, which means that, assuming $\tau_2 = 0$, the unit tax on real balances is positive. However, as we saw in Section 2, when the costs of producing money are zero, $l_m \leq 0$, the nominal interest rate is always positive. A negative inflation tax is compatible with a positive nominal interest rate since $\tau_m \equiv I_t/p_{mt} - 1$.

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7 Since we assume that $l_m \leq 0$, the nominal interest rate is always positive. A negative inflation tax is compatible with a positive nominal interest rate since $\tau_m \equiv I_t/p_{mt} - 1$. 

the optimal unit tax on real balances is zero. The intuition for this result is that the limit of the sequence of optimal unit taxes, when the costs of producing real balances approach zero, also converges to zero, due to the dominant effect of the production costs. In this sense, the result of Proposition 1 can be interpreted as the limit of the result of Proposition 2.

4. Understanding the inflation tax results

In this section we provide the intuition for the results obtained in Sections 2 and 3 for the optimal taxation rules of real balances. We have seen that when money is a free good, the optimal inflation tax is zero for transactions costs functions of any degree of homogeneity. If instead money requires resources to be produced the optimal tax on real balances depends on the degree of homogeneity of that function. If the function is CRS, and the tax on labor used in the production of money is zero, then again the optimal inflation tax is zero.

The result that intermediate goods should not be taxed in a second-best environment, when the technology is CRS, is well known since the work of Diamond and Mirrlees (1971). They show that efficiency in production is a characteristic of the second-best solution when taxes on consumption are available. As a corollary of this result intermediate goods should not be taxed in that case.

Efficiency in production means that labor is allocated optimally, as in the first-best, among alternative uses. Therefore it implies that the marginal productivity of labor used to produce a certain good has to be identical to the marginal productivity of the intermediate good used in the production of the same good times the marginal productivity of labor used to produce the intermediate good. In the set-up of the monetary model in this paper, where there is one composite consumption good and no capital, the optimal consumption tax proposed by Diamond and Mirrlees (1971) is equivalent to a uniform tax on labor: a single ad-valorem rate on labor income generated in every sector and a zero tax rate on intermediate goods.8

The Diamond and Mirrlees (1971) taxation rules, however, do not apply directly to the monetary economy because this is characterized by a specific structure of production and by restrictions on the available set of taxes. The main aspects of this distinctive structure are, first, that the consumption good requires both units of time and transactions, according to a Leontief production function, and, second, that the time used for transactions, and transactions themselves, cannot be taxed. In this context, efficiency in production is attained by abstaining...

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8 There are of course alternative tax structures to this one, in the sense that they support the same allocation: It is straightforward to see that productive efficiency can also be obtained by taxing at different rates labor used in two consecutive stages of production and by taxing (or subsidizing) the intermediate good used in the upper stage.
from taxing labor used to produce real balances, as well as real balances themselves, and taxing only production time. If the transactions costs function is CRS, then it is optimal to have efficiency in production, and therefore real balances should not be taxed. If however, the function is not CRS, then, in this particular context, it is optimal to distort production, and the optimal ad-valorem tax rate on real balances will not be zero (for a zero tax rate on time used to produce real balances). As the cost of producing real balances approaches zero, the unit tax on real balances also approaches zero. So, the result on the optimality of the Friedman rule is ultimately due to the free good characteristic of real balances.

In order to fully understand the results in Sections 2 and 3, we construct fictitious real economies, where the taxation rules of free and intermediate goods are determined. We first consider the simplest structure where we can compute the optimal taxation rules of an intermediate good, for homogeneous production functions, as well as the rules for a free good. So, there are one or two stages of production depending on the relevant input being a free good or an intermediate good. Subsequently we construct a more complex real economy that reproduces the special structure of taxation and production in the monetary model. In both environments, we start by deriving the optimal rules for the ad-valorem tax on the intermediate good when its costs of production are positive and, in a second stage deduce the optimal unit tax when these costs are zero, and so, the good is a free good.

In a fictitious real economy, preferences are defined over consumption, \( c \), and leisure, \( h \). \( c \) is produced using a good, \( m \), and labor, \( n_1 \), according to a production function, \( c = f(m, n_1) \). Alternatively, to be consistent with the specification in the previous sections, the production function can be defined as \( n_1 = l(c, m) \). \( m \) is produced using labor, \( n_2 \), at a constant marginal rate \( \alpha (m = \alpha n_2) \). There is a tax on \( n_1 \), \( \tau_1 \), a tax on \( n_2 \), \( \tau_2 \), and an ad-valorem tax on \( m \), \( \tau_m \). The Ramsey problem is described as follows: Choose \( c \), \( h \), and \( m \) to maximize \( U(c, h) \) such that the implementability condition,

\[
U_c c - U_h [1 - h] - U_c \Pi(c, m) = 0,
\]

and the resources constraint,

\[
l(c, m) + h + m/\alpha \leq 1 - g,
\]

are satisfied. \( \Pi \) are profits obtained in the competitive production of \( c \), that are in general a function of \( c \) and \( m \).

The second-best marginal condition for \( m \) is

\[
-\psi U_c \Pi_m + \lambda (l_m + 1/\alpha) = 0,
\]

where \( \psi \) and \( \lambda \) are the Lagrange multipliers of the implementability and the resources constraints, respectively. From the first-order conditions of the private

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Footnote: See the Appendix for a detailed treatment of these models.
problem, we obtain that there is one degree of freedom in the choice of the taxes. To tax labor used in the production of \( m \) is equivalent to tax \( m \) directly. The marginal conditions of the private problem require that

\[
-l_m = \{(1 - \tau_1)(1 + \tau_m)\}/\{(1 - \tau_2)\alpha\}.
\]

The second-best results are the following: If the production function is CRS, then profits are zero, and the second-best marginal rule is \(-l_m = 1/\alpha\). Surprisingly it turns out that the same result holds for production functions of any degree of homogeneity. If \( \Pi \) is not zero, then the optimal tax on \( m \) depends on the partial effect of \( m \) on profits. It turns out that for any homogeneous function \( f \), profits can be written as \( \Pi = (1 - d)c \), where \( d \) is the degree of homogeneity of \( f \). In this case the partial effect of \( m \) on profits is zero and it is optimal to set \(-l_m = 1/\alpha\). This is the condition of efficiency in production since, from the implicit function theorem \( l_m = -f_m/f_n \), and so \( f_m \alpha = f_n \). The optimal tax on \( m \) depends on the tax on labor used in that sector, \( \tau_2 \). There are multiple choices of taxes that generate the second-best allocation. If \( \tau_2 = \tau_1 \), then \( \tau_m = 0 \). So if labor is taxed at the same rate in both sectors, it is optimal not to tax the intermediate good. This is of course the result of Diamond and Mirrlees (1971) for CRS production functions. We show that in this simple environment, their results can be extended to production functions of any degree of homogeneity.

Since the unit tax on money, \( T_m \), is defined as \( T_m = \tau_m p_m \), we get that for any degree of homogeneity of the production function and for any cost of producing \( m \), the optimal unit tax is zero. If the assumption was that \( \tau_2 = 0 \), then the optimal ad-valorem tax rate on the intermediate good would be \( \tau_m = 1/(1 - \tau_1) - 1 \). In the same way, as \( \alpha \) increases, the unit tax approaches zero.

In this simple structure of production the optimal ad-valorem tax on the intermediate good is zero, for any cost of producing it, and therefore the unit tax on the input when that cost of production becomes arbitrarily small is also zero. So, in both cases \( m \) should not be taxed if the production function is homogeneous of any degree (given \( \tau_2 = \tau_1 \), when \( m \) is an intermediate good).

The structure of production in the monetary economy is more complex than the one we have described, and that justifies the results obtained in that environment, when there are costs of producing money (Section 3). In contrast to the simple economy, in the monetary economy, when money is an intermediate good, the optimal inflation tax depends on the degree of homogeneity of the transactions function. We construct now a more complex fictitious real economy (see Fig. 1 for a graphical representation of the production structure). Complexity is increased for three reasons: there is one more stage of production, there are particular restrictions on the tax structure, and the production of the consumption good as a function of transactions and labor is Leontief, so that there is no factors substitutability. Consider then an economy where the agents have preferences defined over consumption, \( c \), and leisure, \( h \). \( c \) is produced using transactions, \( t \),
and labor, $n_1$, according to a Leontief production function, $c = \min(t, n_1)$. The production of $t$ requires time, $s$, and $m$. As before $m$ is produced with labor, $n_2$, at a constant marginal rate ($m = \alpha n_2$). There is one unit of time in the economy. The tax structure in this economy is such that $c$, $n_1$, $n_2$, and $m$ can be taxed, but $t$ and $s$ cannot be taxed. These are the natural restrictions on taxes in the monetary economy (transactions are produced in the household).

The resources constraint in this economy is $n_1 + n_2 + s + h \leq 1 - g$. Since the production of $c$ is Leontief in $t$ and $n_1$, with unitary coefficients, and $s = l(t, m)$ the resources constraint can be written as

$$c + l(c, m) + h + m/\alpha \leq 1 - g. \quad (4.1)$$

The implementability condition is described by

$$U(c, h) - U(h) [1 - h] - U(c, \Pi) = 0, \quad (4.2)$$

where $\Pi$ is the level of profits in the production of $t$, in units of $c$. The Ramsey problem is to choose $c$, $h$, and $m$ to maximize $U(c, h)$ such that the implementability condition, (4.2), and the resources constraint, (4.1), are satisfied.

From the first-order condition for $m$ we have

$$-\psi U_c \Pi_m + \lambda (l_m + 1/\alpha) = 0, \quad (4.3)$$

where $\psi$ and $\lambda$ are the shadow prices of (4.2) and (4.1), respectively. From the competitive equilibrium conditions, we have again that $-l_m = \{(1 - \tau_1)(1 + \tau_m)\}/\{(1 - \tau_2)\alpha\}$. The second-best results for the optimal tax on $m$ are the following: If the production function is CRS, $\Pi$ is zero and, as before, the second-best marginal rule is $l_m + 1/\alpha = 0$, so that the optimum is characterized by efficiency in production. If $\Pi$ is not zero, then the optimal tax on $m$ depends on the partial effect of $m$ on profits. When the function $f$ is homogeneous, profits in units of $t$
can be written as $\Pi' = (1-d)c$. However, $\Pi$ is measured in units of consumption and so $\Pi = p_t \Pi'$, where $p_t$ is the price of $t$ in units of the consumption good $c$. In this particular structure $p_t$ depends on $m$. From utility and profit maximization for given prices we have

$$p_t f_s = w_s = U_h/U_c,$$

and then $p_t = (U_h/U_c)(1/f_s)$. The expression for profits $\Pi$ is the following:

$$\Pi = \frac{U_h}{U_c} \frac{1}{f_s} (c - f_m m - f_s s) = \frac{U_h}{U_c} (cl_c + l_m m - s), \quad (4.4)$$

since $1/f_s = l_c$ and $l_m = -f_m/f_n$. In the particular structure of production and taxation rules that characterizes this economy, the relevant condition on the production function is the homogeneity of the function $l$: In expression (4.4), real profits are expressed in units of the consumption good, in the term in parentheses on the left-hand side, and are converted into units of time in the term in parentheses on the right-hand side. If, as in the equivalent monetary economy, $l$ is homogeneous of degree $k$, then profits can be written as

$$\Pi = \frac{U_h}{U_c} [k - 1] l(c, m). \quad (4.5)$$

The implementability condition is expressed as

$$U_c c - U_h [1 - h] - U_h [k - 1] l(c, m) = 0. \quad (4.6)$$

This is exactly the expression obtained in Section 2. In Section 2, the production of transactions is in the household and so the meaning of the third term in the expression was not made clear. It amounts to the implicit profits made in the production of transactions. The optimal rule (4.3) can be written as

$$-\psi U_h (k - 1) l_m + \lambda (l_m + 1/\alpha) = 0,$$

and then it is optimal to set

- $l_m = 1/\alpha$, when $k = 1$,
- $l_m > 1/\alpha$, when $k > 1$,
- $l_m < 1/\alpha$, when $k < 1$.

Using the conditions of the private problem, and setting $\tau_2 = 0$, we have that the second-best taxation rules are the following:

- $\tau_m = 0$, when $k = 1$,
- $\tau_m > 0$, when $k < 1$,
- $\tau_m < 0$, when $k > 1$.

In this case it is optimal to keep efficiency in the production of $t$, ($-l_m = 1/\alpha$), only when the production function of $t$ is CRS. When there are profits, the
effect of taxes on profits explains the deviations from efficiency in production in the second-best solution. For the case of an homogeneous function $l$, when $k < 1$, profits are negative and a decrease in $m$ has a negative marginal effect on profits. This explains why it is optimal to tax $m$ allowing for inefficiency in production. When $k > 1$, it is an increase in $m$ that reduces profits. When $k \neq 1$, the possibility of nonzero profits and the absence of a tax on these justifies optimal taxation rules that induce a reduction in profits. The decrease in profits (even negative ones) is equivalent to a lump-sum tax. So the second-best solution allows for a distortion in production that trades off with the lump-sum effect of the indirect taxation of profits. The government can optimally choose to tax intermediate goods and break productive efficiency, when by doing so there are negative effects on profits in the sector where $t$ is produced.

The reason why efficiency in the production of $t$ is attained when both $\tau_2$ and $\tau_m$ are zero is due to the fact that $\tau_e$ is restricted to be equal to zero. Then, by abstaining from taxing both the labor used in the production of $m$, and $m$ itself, that branch of production of transactions is not distorted. Since transactions and time are used in fixed proportions to produce the consumption good, the production is not distorted by the tax on $n_1$. So, the Ramsey solution, even with the particular restrictions on the tax structure ($s$ and $t$ cannot be taxed) is a second-best, and not a third- or fourth-best. If $\tau_2$ was restricted to be equal to $\tau_1$, efficiency in production would require $\tau_m$ to be negative. Therefore we can say that the result obtained when the transactions technology is CRS, that $m$ should not be taxed, is a guaranty of efficiency in production but is not a natural extension of the Diamond and Mirrlees result, given the particular tax restrictions. In this case the intermediate good is not taxed but labor income is taxed at a very discriminatory rate between sources: labor income in the production of money and in the production of transactions is taxed at a zero rate and labor income in the production of the consumption good is taxed at a positive rate.

When $m$ is a free good, the marginal condition of the Ramsey problem for $m$ is

$$-\psi U_c \Pi_m + \lambda l_m = 0,$$

where $\Pi_m = (U_h/U_c)(k - 1)l_m$, from (4.5). At the point of satiation in real balances ($l_m = 0$), the marginal effect of $m$ on profits is zero, and so that point solves the equation. In spite of the fact that, for homogeneous transactions costs functions of degree of homogeneity $k \neq 1$, the level of implicit profits is different from zero, and that in general there is a marginal effect of $m$ on profits, at the satiation point in real balances, where the free good has zero marginal productivity, this effect is null and therefore the point of satiation in real balances is the optimum quantity of money. This result can also be seen as a limiting result of the optimal unit tax on a costly intermediate good, as the cost of producing the good becomes arbitrarily small. The intuition is that the unit tax that is equivalent to a finite ad-valorem tax on a good with an arbitrarily small
cost of production is arbitrarily small. The conclusion is that the optimality of the Friedman rule under general conditions is directly linked to the assumption that the costs of supplying money are negligible.

In any case all that is required for the general result of optimality of the Friedman rule to hold is that the assumption of no resources required for the production of money applies to the variable costs. We take it for granted that while it is evident that there are significant fixed costs of supplying money the same is not true regarding variable costs. So, money as a free primary input, rather than an intermediate good, is the quantitatively relevant assumption and also the theoretical main justification of the generality of an optimal Friedman rule.

5. Conclusions

In this paper we show that the optimality of the Friedman rule, i.e., a zero nominal interest rate, is a general result in monetary models with homogeneous transactions costs functions and where alternative taxes are distortionary. The main reasons for this result are that money has the characteristics of a free primary input and that the inflation tax is (and must be) a unit tax. The generalization of the optimality of the Friedman rule is in contrast with the literature on the optimal inflation tax in similar environments, particularly for the Baumol–Tobin transactions costs functions.

The second main conclusion of the paper is that the existing results on intermediate goods taxation rules in the public finance literature cannot be directly applied in this framework, both when money is costless to produce and when it requires resources for its production. The reason why that is so, in the case with costs of producing money, is that the monetary economy is characterized by a particular structure of production and a specific tax code (transactions cannot be taxed since they are produced in the household). We show that, in this case, the optimal tax on real balances depends on the degree of homogeneity of the transactions costs functions. When this function is CRS, then efficiency in production should hold. This requirement of efficiency in production, in this structure where time used in the production of transactions cannot be taxed, is satisfied with a zero tax on real balances and a zero tax on labor used in the production of money. So, although the result of efficiency in production of Diamond and Mirrlees (1971) holds here as well, the corresponding optimal taxation rules are distinct, due to the particular characteristics of this environment. If the transactions function is not constant returns to scale, then it is optimal for production to be distorted, with the purpose of taxing profits. In this case, if labor used in the production of money is not taxed, the optimal inflation tax is not zero.

When the costs of producing money are zero, so that real balances are a free good, as they are normally taken to be, then the Diamond and Mirrlees (1971)
taxation rules cannot be applied, simply because these are rules on ad-valorem taxes applied on costly goods. When the cost of producing a good is very small, even though the optimal ad-valorem tax may be positive, the optimal unit tax may be also very small, as it is in these monetary environments.

Appendix

A.1. Proof of Proposition 1

We start by writing the first-order conditions of the private problem. We then proceed to describing the Ramsey problem and establishing the Ramsey solution. The following conditions are first-order conditions of the private problem formalized in Section 2.2, for \( t \geq 0 \),

\[
\frac{U_c(t)}{U_h(t)} = \frac{1}{1 - \tau_t} + I_c(t), \tag{A.1}
\]

\[
\frac{U_h(t)}{\beta U_h(t + 1)} = (1 + r_t) \frac{1 - \tau_t}{1 - \tau_{t+1}}, \tag{A.2}
\]

\[- I_m(t) = \frac{1}{1 - \tau_t} I_t, \tag{A.3}
\]

where \( 1 + r_t = (1 + i_t) p_t / p_{t+1} \) and \( I_t = i_t / (1 + i_t) \), \( 0 \leq I_t \leq 1 \).

Note that, in Eq. (A.1), the marginal cost of the consumption good is composed of two terms, the production cost and the term, \( I_c \), that reflects the fact that in order to consume one unit of \( c \), the agents must pay the time cost of transactions, for a given quantity of real balances.

Let \( d_t = 1 / ((1 + r_0) \ldots (1 + r_{t-1}) \ldots) \), with \( d_0 = 1 \). Since at the optimum, \( \lim_{t \to \infty} (d_t m_t + d_t B_t / p_t) = 0 \), we can write the set of budget restrictions as

\[
\sum_{t=0}^{\infty} d_t c_t + \sum_{t=0}^{\infty} d_t l_t m_t = \sum_{t=0}^{\infty} d_t (1 - \tau_t)(1 - h_t - s_t). \tag{A.4}
\]

The Ramsey problem, formalized in Section 2.3, is defined as the choice of \( \{c_t, h_t, m_t\}_{t=0}^{\infty} \) to maximize welfare subject to the implementability condition, (A.5), and the resources constraints, (A.6). The following conditions are necessary conditions for an interior solution of the Ramsey problem, for \( t \geq 0 \),

\[
\sum_{t=0}^{\infty} \beta^t [U_c(t) c_t - U_h(t)(1 - h_t) + U_h(t)(1 - k) l(t)] = 0, \tag{A.5}
\]

\[
c_t + g_t \leq 1 - h_t - l(c_t, m_t). \tag{A.6}
\]

\[
\beta^t U_c(t) + \psi \beta^t [U_c(t) + U_c(t)c_t - U_{h(t)}(1 - h_t + (k - 1)l(t))] + U_h(t)(1 - k) l_c(t)] - [1 + l_c(t)] \lambda_t = 0, \tag{A.7}
\]
Conditions (A.6) with equality, (A.7), (A.8), and (A.9) define $c_t$, $h_t$, $m_t$, $\lambda_t/\beta'$ as functions of $\psi$. These conditions are independent of $t$, and if $g$ is constant over time, the Ramsey solution is stationary in all the variables but for $t$ ($\lambda_t/\beta'$ is stationary).\footnote{The Ramsey solution of this problem is time-consistent. This is so, because the initial total nominal assets are zero and the problem is stationary.}

Eq. (A.9) is satisfied when $l_m(t) = 0$ or $\beta'U_h(t)(1 - k) - \lambda_t = 0$. We start by showing that $\beta'U_h(t)(1 - k) - \lambda_t = 0$ is not a solution of the problem. In order to do so, we replace $l_m(t)$ from (A.9) in (A.8). Using the resulting expression and the implementability condition (A.5), that, since the solution is stationary, becomes $U_c(t)c_t - U_h(t)(1 - h_t - l(t) + kl(t)) = 0$, we obtain the expression

$$U_h(t) + \psi [U_h(t)D_h(t)c_t + U_h(t)k] = 0,$$

where $D(t) = U_c(t)/U_h(t)$ is the marginal rate of substitution, that measures the number of units of leisure, necessary to compensate the agent for a decrease in one unit of consumption. An increase in the quantity of leisure increases this number, as long as consumption is not an inferior good, which is a reasonable assumption, given that consumption is a composite good. So, $D_h(t) > 0$ and therefore the expression in parantheses, in (A.10), is positive. This implies that the equation is satisfied by a negative $\psi$, which means that an increase in the level of government expenditures increases utility, and for the case of $k < 1$, it also implies that the value for the shadow price of resources is negative. The implication of this, is that $\beta'U_h(t)(1 - 1) - \lambda_t = 0$ cannot be a solution of the Ramsey problem. $l_m(t) = 0$ is the only solution that satisfies the first-order conditions with positive multipliers. This solution can be decentralized through a monetary policy characterized by a zero nominal interest rate.

A.2. The dual approach

Guidotti and Végh (1993) use an identical model to the simple monetary model, in Section 2, to claim that the Friedman rule is not a general result. They solve the problem through the dual approach. In fact, since the problem is stationary it can be solved in that way, expressing the quantities as functions of the two taxes. The Ramsey solution is defined as follows:

**Definition 1** (Ramsey solution). The Ramsey solution is a point $(\phi^*,I^*)$ that solves

$$\max_{\phi,I} V(\phi,I) = U[C(\phi,I),H(\phi,I)]$$
s.t.  \( g \leq \phi C(\phi, I) + (1 + \phi)IM(\phi, I) \equiv G(\phi, I) \),

where the functions \( 1 + \phi = 1/(1 - \tau) \) and \( C, H, \) and \( M \) are the private sector solution functions of \( \phi \) and \( I \), i.e., they solve the following system of equations for \( c, h, \) and \( m \), respectively,

\[
U_c(c, h)/U_h(c, h) = 1 + \phi + l_c(c, m), \tag{A.11}
\]

\[-l_m(c, m) = (1 + \phi)l, \tag{A.12}\]

\[(1 + \phi)c + (1 + \phi)lm = 1 - h - l(c, m). \tag{A.13}\]

An interior solution of the Ramsey problem can be described by \( V_c/V_I = G_\phi/G_I \). We check whether this condition is verified at the point \( I = 0 \). At this point, where \( l_m = 0 \), that condition is equivalent to

\[
\phi U_h[H_\phi C_I - H_I C_\phi] = 0. \tag{A.14}\]

Expressions (A.11), (A.12), and (A.13) allow for the determination of the partial derivatives of \( C, H, \) and \( M \) with respect to \( \phi \) and \( I \), at the point \( I = 0 \). When \( m/c \) is finite, that is \( l_{mn}(m/c) \neq 0 \), we obtain the following expressions for those derivatives:

\[
C_\phi = \frac{1 + CD_h}{D_c - DD_h - l_{cc} + l_{cm}^2/l_{mn}}, \tag{A.15}\]

\[H_\phi = -C - C_\phi D, \tag{A.16}\]

\[
C_I = \frac{D_h(1 + \phi)M - l_{cm}(1 + \phi)/l_{mn}}{D_c - DD_h - l_{cc} + l_{cm}^2/l_{mn}}, \tag{A.17}\]

\[H_I = -(1 + \phi)M - DC_I, \tag{A.18}\]

where the function \( D = U_c(C, H)/U_h(C, H) \).

Replacing the expressions (A.15), (A.16), (A.17), (A.18) in (A.14), we obtain

\[
\phi(1 + \phi)U_H l_{mn} \left[ D_c - DD_h - l_{cc} + l_{cm}^2/l_{mn} \right] (k - 1)l_m = 0. \tag{A.19}\]

\( D_c - DD_h - l_{cc} + l_{cm}^2/l_{mn} < 0 \) is a condition of convexity of the private problem. By assumption \( l_{mn} \neq 0 \). At the point \( I = 0 \), \( l_m = 0 \) and so condition (A.19) is verified for any \( k \).

So it is clear that also using this approach the same result of optimality of the Friedman rule for all homogeneous functions is obtained.

A.3. Understanding the inflation tax results

In this section we state the Ramsey problems in two alternative fictitious real economies. The common structure is the following: Preferences are defined over
consumption, $c$, and leisure, $h$. Government expenditures, $g$, are produced with labor with a linear technology with unitary coefficient. Real balances are produced with labor with a linear technology with coefficient $\alpha$. Common notation is: $w_i$ represents wage gross of taxes for labor in sector $i$; $p_j$ represents the price of good $j$. The economies are of two types: Model I, the one/two stages economy, and Model II, that reproduces the monetary economy.

A.3.1. Model I

Consumption is produced using $m$ and labor, $n_1$, according to $c = f(n_1, m)$, or alternatively $n_1 = l(c, m)$. $m$ is produced with labor, $n_2$. There are taxes on labor, $\tau_1$ and $\tau_2$, and a tax on $m$, $\tau_m$. $\Pi$ are the profits in the production of the consumption good. $\Pi^m$ represents profits in the production of $m$, in units of $m$.

Households:

$$\max U(c, h)$$

s.t. 

$$c = (1 - \tau_1)w_{n_1}n_1 + (1 - \tau_2)w_{n_2}n_2 + \Pi.$$ 

Firms:

$$\max \Pi = c - w_{n_1}n_1 - p_m(1 + \tau_m)m$$

s.t. 

$$n_1 = l(c, m).$$

$$\max \Pi^m = m - (w_{n_2}/p_m)n_2$$

s.t. 

$$m = \alpha n_2.$$ 

FOC:

$$U_c/U_h = 1/[(1 - \tau_1)w_{n_1}],$$

$$(1 - \tau_1)w_{n_1} = (1 - \tau_2)w_{n_2},$$

$$f_{n_1} = 1/l_c = w_{n_1},$$

$$f_m = -l_m/l_c = p_m(1 + \tau_m),$$

$$w_{n_2}/p_m = \alpha.$$ 

Using the budget constraint and the first condition in the set of FOC, we obtain the implementability condition

$$U_c c - U_h (1 - h) = U_c \Pi.$$ 

The Ramsey problem is defined as the choice of $(c, h, m)$ to maximize $U(c, h)$ subject to the implementability constraint and to the resources constraint

$$l(c, m) + h + m/\alpha \leq 1 - g.$$
The optimal choice of \( m \) in this problem implies that
\[-\psi U_c \Pi_m + \lambda (l_m + 1/2) = 0.\]

A.3.2. Model II

In this model the production function of \( c \) is Leontief, with inputs labor, \( n_1 \), and \( t \). \( t \) is produced with labor, \( s \), and \( m \), according to the production function \( s.t. \ t = f(s,m) \), that can also be described as \( s = l(t,m) \). \( t \) and \( s \) cannot be taxed. \( \Pi \) represents, now, profits in the production of \( t \), measured in units of consumption. \( \Pi' \) represents profits in units of \( t \). Labor can be used in the production of \( m \). \( \Pi^c \) are profits in the production of \( c \), in units of \( c \), and \( \Pi^m \) represents profits in the production of \( m \), in units of \( m \).

**Households:**

\[
\max \ U(c,h) \\
\text{s.t.} \quad c = (1 - \tau_1)w_{n_1}n_1 + w_n s + (1 - \tau_2)w_{n_2}n_2 + \Pi.
\]

**Firms:**

\[
\max \ \Pi^c = c - w_{n_1}n_1 - p_it \\
\text{s.t.} \quad c = \min(n_1,t),
\]

\[
\max \ \Pi' = t - (w_s/p_t)s - p_m(1 + \tau_m)/p_t \\
\text{s.t.} \quad s = l(t,m),
\]

\[
\max \ \Pi^m = m - (w_{n_2}/p_m)n_2 \\
\text{s.t.} \quad m = 2n_2.
\]

**FOC:**

\[
U_c/U_h = 1/[(1 - \tau_1)w_{n_1}],
\]
\[
(1 - \tau_1)w_{n_1} = w_s = (1 - \tau_2)w_{n_2},
\]
\[
c = t,
\]
\[
p_t + w_{n_1} = 1,
\]
\[
f_s = 1/l_c = w_s/p_t,
\]
\[
f_m = -l_m/l_c = p_m(1 + \tau_m)/p_t,
\]
\[
w_m/p_m = \alpha.
\]
Using the budget constraint and the FOC we obtain the implementability constraint, where the profits are expressed as a function of \( c \) and \( m \),

\[
U_c c - U_h [1 - h] = U_c \Pi(c, m).
\]

The Ramsey problem is defined as the choice of \((c, h, m)\) to maximize \( U(c, h) \) subject to the implementability constraint and to the resources constraint

\[
l(c, m) + h + m/\alpha \leq 1 - g.
\]

The optimal choice of \( m \) in this problem implies that

\[
-\psi U_c \Pi_m + \lambda (l_m + 1/\alpha) = 0.
\]

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