On the relevance of exchange rate regimes for stabilization policy

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Abstract
This paper assesses the relevance of the exchange rate regime for stabilization policy. Using both fiscal and monetary policy, we conclude that the exchange rate regime is irrelevant. This is the case independently of the severity of price rigidities, independently of asymmetries across countries in shocks and transmission mechanisms. The only relevant conditions are on the mobility of labor and financial assets. The results can be summarized with the claim that every currency area is an optimal currency area. However, with labor mobility or tradable state-contingent assets, additional policy instruments would be required to establish the irrelevance result.

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1. Introduction
This paper revisits the issues in the optimal currency area literature initiated by Mundell [24]. What are the costs of a fixed exchange rate regime, or a monetary union, when there is a role for stabilization policy? We address this question allowing for heterogeneity in the shocks and the
response to them, restrictions on the mobility of factors and incompleteness of asset markets, as is standard in the optimal currency area literature.

When different shocks hit different countries or when there are differences across countries in the effects of shocks, monetary policy, that has a stabilization role because of some form of nominal rigidity, may have to react differently in the different countries. Because of this heterogeneity it is common to infer that there are costs of a common monetary policy, either through a fixed exchange rate regime or a monetary union. In the literature, these costs are taken to be higher the stronger are the asymmetries, the more severe are the nominal rigidities, the more pronounced is the incompleteness of international asset markets, the less mobile is labor, and, finally, the less able is fiscal policy in effectively stabilizing the national economies (Corsetti [12]).

We take the standard approach in the literature on optimal fiscal and monetary policy after Lucas and Stokey [23], followed by many others. There, fiscal and monetary policy are decided jointly by a Ramsey government that must raise distortionary taxes to pay for exogenous government expenditures, so that the Pareto first best solution is not achievable. In our second best environment, we show that the loss of the country specific monetary tool is of no cost. This is true irrespective of the asymmetry in shocks or response to these and the severity of the nominal rigidities. The elements that are crucial in assessing the costs of a single monetary policy are the three last ones in the list by Corsetti above, but labor mobility, and the completeness of international financial markets, work in opposite ways to the conventional wisdom. Fiscal and monetary policy are able to eliminate the costs of a monetary union only if labor is not mobile across countries and private state-contingent debt is not traded internationally. Unless further instruments are considered.

We consider a standard two country model. Each country specializes in the production of a composite tradable good, which aggregates a continuum of goods produced using labor only. Labor is not mobile across countries. Money is used for transactions according to a cash-in-advance constraint on the purchases of the two composite goods by the households of each country. The government of each country must finance exogenous expenditures on the good produced at home with distortionary taxes and seigniorage. The tax instruments are labor income and consumption taxes. There is state-contingent private debt inside each country in zero net supply and non-contingent nominal public debt in each currency that can be traded internationally.

We start by analyzing a benchmark economy where prices are flexible (Sections 2 and 3). We show that any equilibrium allocation in the flexible price, flexible exchange rates, economy can be implemented with fiscal and monetary policies that induce stable producer prices and constant exchange rates. This result has implications for economies under fixed exchange rates with nominal rigidities (Section 4). For those policies, that under flexible prices keep prices constant, if firms were restricted in the setting of prices such as in Calvo [7], those restrictions would be irrelevant and the same allocations could still be implemented. It follows that under sticky prices and fixed exchange rates it is always possible to achieve the same allocations as under flexible prices and exchange rates.

Under sticky prices there are equilibrium allocations other than the ones achieved under flexible prices. We show that the common set of allocations to flexible and sticky prices dominates in welfare terms those other equilibrium allocations. The reason for this result is the one in Diamond and Mirrlees [16], that even in a second best environment it is not optimal to distort production.

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1 See also Corsetti [13].
2 Lump-sum taxes are excluded for distribution reasons. If there were both distortionary and lump-sum taxes, the irrelevance results would still be obtained. It would be possible, in that case, to attain the first best allocations.
With the two results, (i) that under fixed exchange rates and sticky prices it is possible to implement the set of allocations under flexible exchange rates and flexible prices and (ii) that the set of allocations under flexible prices is optimal, we are able to establish that the choice of the exchange rate regime is irrelevant for optimal stabilization policy.

When prices are sticky, one would think that flexible exchange rates would be useful in adjusting the relative price of goods to different shocks. In our model, because we allow for consumption taxes, the relative price is the ratio of prices gross of consumption taxes, adjusted by the nominal exchange rate. When the exchange rate is fixed, consumption taxes have a direct effect on the relative price and move in response to shocks so that the necessary adjustments take place. Labor income taxes also have to be adjusted so that other margins are not affected.

Exchange rate policy can play other roles such as completing the noncontingent international financial markets. When the exchange rates are fixed, taxes, and interest rates common across countries, play that same role of allowing for the returns on assets traded internationally to vary with the shocks.

Labor immobility is an important assumption for our results (Section 5.1.1). Labor mobility imposes additional equilibrium restrictions, in particular arbitrage conditions on the choice of where to work, that cannot be satisfied with the policy instruments that we consider. Similarly, perfect capital mobility would also require further instruments, for the irrelevance results to hold (Section 5.1.2). These results go against the traditional claims in the optimal currency area literature.

Related literature reassesses Milton Friedman [20]'s case for exchange rate flexibility, as a way of side-stepping the rigidity in relative price movements. Recent examples in the debate are, for instance, Devereux and Engle [15] and Duarte and Obstfeld [18]. Devereux and Engle [15] provide an example with local currency pricing where exchange rate flexibility is of no use. Because the prices of goods are set in the currency of the consumers, the exchange rate cannot affect the relative price. Duarte and Obstfeld [18] respond, showing that exchange rate flexibility can still be of use in a more complex environment with nontradable goods. Even if exchange rate movements cannot affect the relative price of goods, they can still affect the allocations and improve welfare. Because the optimal exchange rate regime depends on the degree of exchange rate pass-through, Corsetti and Pesenti [14] endogenize the decision on which currency prices are set in. They show that there are two self validating regimes, one with fixed and another with flexible exchange rates. The flexible exchange rate regime provides higher welfare. Our paper questions the generality of the exercises in these papers. We show that the claims hinge on the focus on monetary policy only. Once the choice of the exchange rate regime is considered in the context of the full choice of policy instruments including tax and debt policy, exchange rate flexibility can be replaced with a gain by fiscal instruments. In the set up of Devereux and Engle [15], the exchange rate regime would still be irrelevant, but it would be possible to implement better allocations; the remark of Duarte and Obstfeld [18] would not go through; and the two regimes in Corsetti and Pesenti [14], flexible or fixed exchange rates, would provide the same welfare.

Cooper and Kempf [10] make a similar point to ours in a very different context. They explicitly model the Mundellian trade-off between the benefits of a monetary union in reducing transaction

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3 See Chari, Christiano and Kehoe [8], Schmitt-Grohé and Uribe [27], Siu [28], Correia, Nicolini and Teles [11], and Aiyagari, Marcet, Sargent and Seppala [1] for optimal policy without state contingent public debt in a closed economy. See also Angeletos [2], and Buera and Nicolini [6], on the use of debt of different maturities as a way of completing asset markets.

4 For recent work on optimal monetary policy in a currency area see Benigno [4].
costs and the costs of the union in the ability to stabilize. Stabilization in their set up are risk sharing transfers between agents. If the government is able to stabilize using alternative fiscal instruments, then there are no costs of a monetary union.

Gali and Monacelli [21] and Ferrero [19] consider both fiscal and monetary policy in a set up closer to ours, but restrict the set of fiscal policy instruments. For that reason, they are unable to establish the irrelevance results that we obtain. In Gali and Monacelli [21] the government chooses the optimal level of public consumption in a monetary union with lump-sum taxes. The use of state-contingent public consumption is useful, but is not a substitute for exchange rate flexibility. Ferrero [19], like us, considers that lump-sum taxes are not available. He solves a very similar policy problem to ours with the main difference that consumption taxes are not considered. He allows for state-contingent labor income taxes, but not for consumption taxes. As it turns out, that assumption is crucial. While we are able to establish that exchange rate flexibility is irrelevant, in Ferrero [19] all fiscal policy does is help attain higher welfare. There is still a cost of a monetary union.

We assume that both fiscal and monetary policy variables are state-contingent. Because our question is a regime question, we think that the natural assumption is to allow for alternative fiscal and monetary institutions, other than the ones we observe. This was the approach in Lucas and Stokey [23], followed by the subsequent literature (see Chari, Christiano and Kehoe’s multiple contributions, Correia, Nicolini and Teles [11], Schmitt-Grohé and Uribe [27], Siu [28], Benigno and Woodford [5], and, in the open economy, Benigno and Paoli [3], Ferrero [19] among others). We do not think there are fundamental reasons for taxes not to be state-contingent. If there were, then they should probably also apply to monetary policy, given that monetary policy acts essentially like fiscal policy, in the models that we use. But since we do observe that monetary policy is very flexible, most likely, there is nothing preventing fiscal policy from being as flexible.

The results and methodology in Correia et al. [11] are instrumental for the results we obtain here. Correia et al. show, in the closed economy, that optimal allocations and policies do not depend on whether prices are flexible or sticky. In this paper, we show that the results hold in the open economy, as well, irrespective of the degree of price rigidity and of the form, whether prices are set in the producer or the consumer currency. But, more important, we show that sticky prices are irrelevant when exchange rates are fixed. This is not only counter intuitive, but it is also contrary to the existing results in the literature. It allows us to contribute to the literature on optimal currency areas with a stark irrelevance result.

2. The model

The economy has two countries of equal size, the home country and the foreign country. In each country there is a representative household, a continuum of firms and a government. Each firm produces a distinct, perishable consumption good with labor only. In each period \( t = 0, 1, \ldots, T \), where \( T \) can be made arbitrarily large, the economy experiences one of finitely

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5 There is related work on optimal fiscal and monetary policy in small open economies. In Nicolini and Hevia [25], even if prices are sticky, the second best, flexible price equilibrium is implementable, but exchange rates must move across states. See also Benigno and Paoli [3].

6 See, in particular, Chari, Christiano and Kehoe [8] and Chari and Kehoe [9].

7 The assumption of a finite, even if arbitrarily large, time horizon considerably simplifies the analysis, and is as reasonable an assumption as the more standard one of an infinite horizon.
many events \( s_t \). The initial realization \( s_0 \) is given. The set of all possible events in period \( t \) is denoted by \( S_t \), the history of these events up to and including period \( t \), which we call state at \( t \), \((s_0, s_1, \ldots, s_t)\), is denoted by \( s^t \), and the set of all possible states in period \( t \) is denoted by \( S^t \). The number of all possible states in period \( t \) is \#\( S^t \). All the relevant variables for this world economy are a function of the state but to simplify the notation we do not index formally the variables to the state.

There are markets for goods, labor, money, state-contingent debt and state-noncontingent debt. The labor market is segmented across countries. The state-contingent debt market is segmented across countries and across households and governments. The goods and the state-noncontingent debt are tradable across countries and agents. In this section we assume that firms set prices every period with contemporaneous information. We also assume that exchange rates are flexible.

2.1. The households

The preferences of the home households are described by the expected utility function

\[
U = E_0 \sum_{t=0}^{T} \beta^t u(C_{h,t}, C_{f,t}, L_t). \tag{1}
\]

\( C_{h,t} \) is the home composite consumption good that aggregates the goods produced by the home firms,

\[
C_{h,t} = \left[ \int_0^1 C_{h,t}(i)^{\frac{\theta - 1}{\theta}} di \right]^\frac{\theta}{\theta - 1}, \quad \theta > 1, \tag{2}
\]

where \( C_{h,t}(i) \) is the consumption of the good produced by firm \( i \). There is a continuum of home firms indexed by \( i \), in the unit interval. \( C_{f,t} \) is the foreign composite consumption good aggregating the goods produced by the foreign firms,

\[
C_{f,t} = \left[ \int_0^1 C_{f,t}(j)^{\frac{\theta - 1}{\theta}} dj \right]^\frac{\theta}{\theta - 1}. \tag{3}
\]

There is also a continuum of these firms, indexed by \( j \), in the unit interval. \( L_t \) is leisure time and is equal to \( 1 - N_t \), where \( N_t \) is total time devoted to production.

The preferences of the foreign households are described by

\[
U = E_0 \sum_{t=0}^{T} \beta^t u(C_{h,t}^*, C_{f,t}^*, L_t^*),
\]

where \( C_{h,t}^* \) is the foreign households composite consumption of the goods produced in the home country, according to \( C_{h,t}^* = \left[ \int_0^1 C_{h,t}^*(i)^{\frac{\theta - 1}{\theta}} di \right]^\frac{\theta}{\theta - 1} \), and \( C_{f,t}^* \) is the foreign households composite consumption of the goods produced in the foreign country, according to \( C_{f,t}^* = \left[ \int_0^1 C_{f,t}^*(j)^{\frac{\theta - 1}{\theta}} dj \right]^\frac{\theta}{\theta - 1} \).

The households of either country minimize expenditure in the home and foreign goods to obtain a given aggregate consumption of either good. This implies, for the home households,

\[
C_{h,t}(i) = \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} C_{h,t} \quad \text{and} \quad C_{f,t}(j) = \left( \frac{P_{f,t}^*(j)}{P_{f,t}^*} \right)^{-\theta} C_{f,t},
\]

\( P_{h,t}(i) \) and \( P_{f,t}^*(j) \) are the prices of goods produced in the home country and in the foreign country, respectively.
with
\[
P_{h,t} = \left[ \int_0^1 P_{h,t}(i)^{1-\theta} \, di \right]^{1/\theta} \quad \text{and} \quad P_{f,t}^* = \left[ \int_0^1 P_{f,t}^*(j)^{1-\theta} \, dj \right]^{1/\theta},
\]
where \(P_{h,t}(i)\) is the price of the good produced by the home firm \(i\) in units of domestic currency, and \(P_{f,t}^*(j)\) is the price of the good produced by the foreign firm \(j\) in units of foreign currency. Expenditure in either composite good purchased by the home households can then be written as
\[
\int_0^1 P_{h,t}(i)C_{h,t}(i) \, di = P_{h,t}C_{h,t} \quad \text{and} \quad \int_0^1 P_{f,t}^*(j)C_{f,t}^*(j) \, dj = P_{f,t}^*C_{f,t}.
\]

Similar expressions are obtained for the households of the foreign country.

The budget constraints can then be written in terms of the aggregate variables. The representative household of the home country at the beginning of each period \(t = 0, 1, \ldots, T\) uses the nominal wealth \(W_t\) to buy \(M_t\) (home money), \(B_{h,t}\) (home government noncontingent debt), \(B_{f,t}\) (foreign government noncontingent debt) and \(B_{t+1}\) (home private state-contingent debt). The home government noncontingent debt pays the gross return \(R_t\) in the domestic currency at the beginning of the following period, and the foreign government noncontingent debt pays gross return \(R_t^*\) in foreign currency. The price, normalized by the probability of occurrence of the state, at date \(t\) of one unit of domestic currency at a particular state at date \(t+1\) is \(Q_{t,t+1}\). There is no government state-contingent debt and the home household cannot buy foreign contingent debt. The price of one unit of foreign currency in units of home currency is \(\varepsilon_{t}\). Thus, the following restrictions must be satisfied, respectively, for the home and the foreign households,
\[
\begin{align*}
M_t + B_{h,t} + \varepsilon_{t}B_{f,t} + E_{t}B_{t+1}Q_{t,t+1} & \leq W_t, \\
M_t^* + \frac{B_{h,t}^*}{\varepsilon_{t}} + B_{f,t}^* + E_{t}B_{t+1}^*Q_{t,t+1}^* & \leq W_t^*.
\end{align*}
\]

In the home country there are taxes on the consumption of home produced goods, \(\tau_{h,t}\), on the consumption of foreign produced goods, \(\tau_{f,t}\), labor income \(\tau_{n,t}\) and profits. As the tax on profits is lump-sum, it is optimal that all profits be taxed away, so that the net profits are zero.\(^9\) There are corresponding taxes in the foreign country, \(\tau_{h,t}^*, \tau_{f,t}^*, \tau_{n,t}^*\).

The wealth that home and foreign households bring to date \(t+1\) is, respectively,
\[
\begin{align*}
W_{t+1} &= M_t - (1 + \tau_{h,t})P_{h,t}C_{h,t} - (1 + \tau_{f,t})\varepsilon_{t}P_{f,t}^*C_{f,t} \\
& \quad + B_{h,t}R_t + \varepsilon_{t}B_{f,t}R_{t}^* + B_{t+1} + (1 - \tau_{n,t})W_tN_t, \\
W_{t+1}^* &= M_t^* - (1 + \tau_{h,t}^*)\frac{P_{h,t}}{\varepsilon_{t}}C_{h,t} - (1 + \tau_{f,t}^*)P_{f,t}^*C_{f,t}^* \\
& \quad + \frac{B_{h,t}^*}{\varepsilon_{t+1}}R_t + B_{f,t}^*R_{t}^* + B_{t+1}^* + (1 - \tau_{n,t}^*)W_t^*N_t^*.
\end{align*}
\]

\(^8\) We assume that there is an additional subperiod at \(T+1\) with an assets market for the clearing of debts, which guarantees that money has value in the finite horizon economy. Agents want to take money to period \(T+1\) to settle debts. If the finite horizon economy ended with a goods market at \(T\), then sellers would not accept money in period \(T\), and therefore money would not have value, not only in that period but in every period.

\(^9\) This assumption that profits are fully taxed is without loss of generality.
Money is used to purchase goods according to the following cash-in-advance constraints, for the home and foreign country, respectively,

\[
(1 + \tau_{h,t})P_{h,t}C_{h,t} + (1 + \tau_{f,t})\varepsilon_t P_{f,t}^* C_{f,t} \leq M_t, \quad \text{all } s^t, \ 0 \leq t \leq T, \quad (6)
\]

\[
(1 + \tau_{h,t}^*)P_{h,t}^* C_{h,t}^* + (1 + \tau_{f,t}^*)P_{f,t}^* C_{f,t}^* \leq M_t^*, \quad \text{all } s^t, \ 0 \leq t \leq T. \quad (7)
\]

The households of the home country take prices, policies and initial wealth as given and choose allocations and asset positions that maximize expected utility (1) subject to the cash-in-advance constraints (6) and the budget constraints (4) and (5) for the home country, together with \(W_{T+1} \geq 0\). The households of the foreign country solve a similar problem.

Among the first order conditions for the home and foreign households are the intertemporal conditions for the contingent assets,

\[
Q_{t-1,t} \frac{u_{C_h}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta R_{t-1}E_{t-1} \left[ \frac{u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})} \right], \quad \text{all } s^t-1 \text{ and } s^t | s^{t-1}, \ 1 \leq t \leq T, \quad (8)
\]

\[
Q_{t-1,t}^* \frac{\varepsilon_{t-1}u_{C_h}^*(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta \varepsilon_t R_{t-1}E_{t-1} \left[ \frac{\varepsilon_t u_{C_h}^*(t)}{P_{h,t}(1 + \tau_{h,t}^*)} \right], \quad \text{all } s^t-1 \text{ and } s^t | s^{t-1}, \ 1 \leq t \leq T, \quad (9)
\]

the intertemporal conditions for the noncontingent assets,

\[
\frac{u_{C_h}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta R_{t-1}E_{t-1} \left[ \frac{u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})} \right], \quad \text{all } s^t-1, \ 1 \leq t \leq T, \quad (10)
\]

\[
\frac{\varepsilon_{t-1}u_{C_h}^*(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta \varepsilon_t R_{t-1}E_{t-1} \left[ \frac{\varepsilon_t u_{C_h}^*(t)}{P_{h,t}(1 + \tau_{h,t}^*)} \right], \quad \text{all } s^t-1, \ 1 \leq t \leq T, \quad (11)
\]

\[
\frac{u_{C_h}^*(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta R_{t-1}E_{t-1} \left[ \frac{u_{C_h}^*(t)}{P_{h,t}(1 + \tau_{h,t})} \right], \quad \text{all } s^t-1, \ 1 \leq t \leq T, \quad (12)
\]

\[
\frac{\varepsilon_{t-1}u_{C_h}^*(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta \varepsilon_t R_{t-1}E_{t-1} \left[ \frac{\varepsilon_t u_{C_h}^*(t)}{P_{h,t}(1 + \tau_{h,t}^*)} \right], \quad \text{all } s^t-1, \ 1 \leq t \leq T. \quad (13)
\]

and the intratemporal conditions,

\[
\frac{u_L(t)}{u_{C_h}(t)} = \frac{W_t(1 - \tau_{n,t})}{P_{h,t}R_t(1 + \tau_{h,t})}, \quad \text{all } s^t, \ 0 \leq t \leq T, \quad (14)
\]

\[
\frac{u_{C_h}(t)}{u_{C_f}(t)} = \frac{(1 + \tau_{h,t})P_{h,t}}{(1 + \tau_{f,t})\varepsilon_t P_{f,t}^*}, \quad \text{all } s^t, \ 0 \leq t \leq T. \quad (15)
\]

\[
\frac{u_{L_f}(t)}{u_{C_f}(t)} = \frac{W_t^* (1 - \tau_{n,t}^*)}{P_{f,t}^* R_t^*(1 + \tau_{f,t}^*)}, \quad \text{all } s^t, \ 0 \leq t \leq T, \quad (16)
\]

\[
\frac{u_{C_h}^*(t)}{u_{C_f}^*(t)} = \frac{(1 + \tau_{h,t}^*)P_{h,t}^*}{(1 + \tau_{f,t}^*)\varepsilon_t P_{f,t}^*}, \quad \text{all } s^t, \ 0 \leq t \leq T, \quad (17)
\]

The budget constraints of the households of each country, (4) and (5), together with the terminal conditions \(W_{T+1} \geq 0\) and \(W_{T+1}^* \geq 0\), can be written as intertemporal budget constraints, that at the optimum hold with equality. For the home country those constraints are the following:
\[
\sum_{s=t}^{T} \left[ (1 + \tau_{h,s}) P_{h,s} C_{h,s} + (1 + \tau_{f,s}) \epsilon_{s} P_{f,s} C_{f,s} - (1 - \tau_{n,s}) W_{s} N_{s} \right] \\
+ \sum_{s=t}^{T} \left[ M_{s} \left( \frac{Q_{t,s}}{Q_{t,s+1}} - 1 \right) \right] = W_{t}, \quad \text{all } s', \ 0 \leq t \leq T,
\]

where \( Q_{t,s} = Q_{t+1,s} \ldots Q_{s-1,s}, \ 0 \leq t \leq T, \ t + 1 \leq s \leq T + 1, \) and \( Q_{t,t} = 1. \) Also \( E_{T} Q_{T,T+1} = \frac{1}{RT}. \)

Using the marginal conditions, as well as the cash-in-advance constraints, in the intertemporal budget constraints, we can rewrite the household budget constraints in the home country and foreign country, respectively, as

\[
\sum_{s=t}^{T} \left[ (u_{C_{h}}(s) C_{h,s} + u_{C_{f}}(s) C_{f,s} - u_{L}(s) N_{s}) \right] \\
= \frac{u_{C_{h}}(t)}{P_{h,t}(1 + \tau_{h,t})}, \quad \text{all } s', \ 0 \leq t \leq T, \quad (18)
\]

\[
\sum_{s=t}^{T} \left[ (u_{C_{h}^{*}}(s) C_{h,s}^{*} + u_{C_{f}^{*}}(s) C_{f,s}^{*} - u_{L^{*}}(s) N_{s}^{*}) \right] \\
= \frac{u_{C_{f}^{*}}(t)}{P_{f,t}^{*}(1 + \tau_{f,t}^{*})}, \quad \text{all } s', \ 0 \leq t \leq T. \quad (19)
\]

### 2.2. The government

The government of each country includes both the fiscal authority and the monetary authority. We assume, as is standard in this literature, that aggregate public expenditures are exogenous. Each government only consumes goods produced by local firms, and chooses consumption of each good to minimize expenditure on the aggregate level of expenditures, \( G_{t} \) for the home country and \( G_{t}^{*} \), for the foreign country, respectively,

\[
G_{t} = \left[ \int_{0}^{1} G_{h,t}(i) \frac{\theta - 1}{\sigma'} di \right]^{\theta \sigma'},
\]

\[
G_{t}^{*} = \left[ \int_{0}^{1} G_{f,t}^{*}(j) \frac{\theta - 1}{\sigma'} d\sigma \right]^{\theta},
\]

where \( G_{h,t}(i) \) is the home government consumption of the good produced by firm \( i \) and \( G_{f,t}^{*}(j) \) is the foreign government consumption of the good produced in that country by firm \( j \).

The home government issues state-noncontingent debt, \( B_{h,t} + B_{h,t}^{*} \), and money, \( M_{t}^{s} \), and taxes labor income and private consumption, as well as profits. The nominal financial liabilities of the home government at the start of period \( t \) are \( W_{t}^{G} \), which can be financed by issuing money and public debt

\[
M_{t}^{s} + B_{h,t} + B_{h,t}^{*} = W_{t}^{G}.
\]

The nominal financial liabilities the home government brings to the next period are
\[
W_{t+1}^g = M_t^s + R_t B_{h,t} + R_t B_{h,t}^* + P_{h,t} G_t - \tau_{h,t} P_{h,t} C_{h,t} - \tau_{f,t} P_{f,t} C_{f,t}^* - \tau_{n,t} W_t N_t - \Pi_{h,t},
\]
where \(\Pi_{h,t}\) are the aggregate profits of the home firms that are fully taxed. We impose the terminal condition that government liabilities in the terminal period are zero,\(^{10}\) \(W_{T+1}^g = 0\). The home government period \(t\) intertemporal budget constraint can then be written as
\[
\sum_{s=t}^{T} E_t Q_{t,s+1}(\tau_{h,s} P_{h,s} C_{h,s} + \tau_{f,s} P_{h,s} C_{f,s}^* + \tau_{n,s} W_s N_s + \Pi_{h,s} - P_{h,s} G_s)
+ \sum_{s=t}^{T} E_t Q_{t,s+1} M_s^* \left( \frac{Q_{t,s}}{Q_{t,s+1}} - 1 \right) = W_t^g, \quad \text{all } s', 0 \leq t \leq T.
\]

There is a similar condition for the government of the foreign country.

2.3. Firms

In each country there is a continuum of firms in the unit interval. Each firm produces a distinct, perishable consumption good with a technology that uses labor only. Each home firm \(i\) has the production technology
\[
Y_{h,t}(i) = A_t N_t(i), \quad \text{all } s', 0 \leq t \leq T,
\]
where \(Y_{h,t}(i)\) is the production of good \(i\), \(N_t(i)\) is the labor used in the production of good \(i\), and \(A_t\) is an aggregate technology shock in the home country. Good \(i\) can be used for private and public consumption, \(Y_{h,t}(i) = C_{h,t}(i) + C_{h,t}^*(i) + G_t(i)\). The technology in the foreign country is
\[
Y_{f,t}(j) = A_t^* N_t^*(j), \quad \text{all } s', 0 \leq t \leq T,
\]
where the technology parameter \(A_t^*\) is the same across firms but can be different from \(A_t\). Each good \(j\) produced in the foreign country can be consumed by households or by the foreign government, \(Y_{f,t}(j) = C_{f,t}(j) + C_{f,t}^*(j) + G_t^*(j)\).

Prices are flexible. The firms in the home country choose prices to maximize profits \(\Pi_{h,t}(i) = P_{h,t}(i) Y_{h,t}(i) - W_t N_t(i)\), given the demand functions
\[
Y_{h,t}(i) = \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} Y_{h,t}, \quad \text{all } s', 0 \leq t \leq T,
\]
where \(Y_{h,t} = C_{h,t} + C_{h,t}^* + G_t\), obtained using the demand functions of the home good at home and abroad, and given the production functions (20).

The home firms set a common price \(P_{h,t}(i) = P_{h,t}\) such that
\[
\frac{W_t}{P_{h,t}} = \frac{\theta - 1}{\theta} A_t, \quad \text{all } s', 0 \leq t \leq T,
\]
and the foreign firms set \(P_{f,t}^*(j) = P_{f,t}^*\), such that
\[
\frac{W_t^*}{P_{f,t}^*} = \frac{\theta - 1}{\theta} A_t^*, \quad \text{all } s', 0 \leq t \leq T.
\]

\(^{10}\) This results from a no-Ponzi games condition imposed on both households and governments.
2.4. Equilibrium

We now define a flexible price equilibrium as allocations \( \{ C_{h,t}, C_{f,t}, N_t, C^*_{h,t}, C^*_{f,t}, N^*_t \} \) asset positions \( \{ M_t, B_{h,t}, B_{f,t}, B_{t+1}, M^*_t, B^*_{h,t}, B^*_{f,t}, B^*_{t+1} \} \), prices and policies \( \{ P_{h,t}, W_t, R_t, Q_{t,t+1}, \tau_{h,t}, \tau_{f,t}, \tau_{n,t}, M^S_t, \varepsilon_t \} \) and \( \{ P_{f,t}, W^*_t, R^*_t, Q^*_{t,t+1}, \tau^*_{h,t}, \tau^*_{f,t}, \tau^*_{n,t}, M^*_t \} \) for \( t = 0, 1, \ldots, T \) and all \( s' \), such that,

(a) Given the initial wealth levels, prices and policy the households choose the relevant quantities that solve their problems;
(b) Given prices and policy, the firms choose the relevant quantities that solve their problems;
(c) For initial public liabilities the governments satisfy their budget constraints;
(d) The markets are in equilibrium, implying that for all \( s' \), \( 0 \leq t \leq T \),

\[
\begin{align*}
C_{h,t} + C^*_{h,t} + G_t &= A_t N_t, \\
C_{f,t} + C^*_{f,t} + G^*_t &= A^*_t N^*_t, \\
M^S_t &= M_t, \\
M^*_{t} &= M^*_t, \\
B_{t+1} &= 0, \\
B^*_{t+1} &= 0.
\end{align*}
\]

The market clearing in the labor and noncontingent bond markets was already imposed.

The equilibrium conditions that determine the allocations \( \{ C_{h,t}, C_{f,t}, N_t, C^*_{h,t}, C^*_{f,t}, N^*_t \} \), asset positions \( \{ M_t, B_{h,t}, B_{f,t}, B_{t+1}, M^*_t, B^*_{h,t}, B^*_{f,t}, B^*_{t+1} \} \), prices and policies \( \{ P_{h,t}, W_t, R_t, Q_{t,t+1}, \tau_{h,t}, \tau_{f,t}, \tau_{n,t}, M^S_t, \varepsilon_t \} \) and \( \{ P_{f,t}, W^*_t, R^*_t, Q^*_{t,t+1}, \tau^*_{h,t}, \tau^*_{f,t}, \tau^*_{n,t}, M^*_t \} \) are (6)–(19), (22)–(29) and the intertemporal budget constraints of the home country that can be obtained by adding up the home government budget constraints and the home household budget constraints, as follows,

\[
\begin{align*}
\sum_{s=t}^{T} E_t Q_{t,s+1} \left[ P_{h,s} (C_{h,s} + G_s) + \varepsilon_s P^*_{f,s} C_{f,s} - W_s N_s - \Pi_{h,s} \right] &= \mathbb{W}^f_t, \quad \text{all } s', \ 0 \leq t \leq T, \\
\sum_{s=t}^{T} E_t Q_{t,s+1} \left[ \varepsilon_s P^*_{f,s} C_{f,s} - P_{h,s} C^*_{h,s} \right] &= \mathbb{W}^f_t, \quad \text{all } s', \ 0 \leq t \leq T.
\end{align*}
\]

3. Equilibria under flexible prices

Our purpose in this section is to assert a major result of the paper that has implications for equilibrium allocations with sticky prices and fixed exchange rates. We show that the set of equilibrium allocations under flexible prices and flexible exchange rates can be implemented with policies such that the price level in either country is constant over time, and such that the nominal exchange rate is also constant over time. In order to do this we show that, for a given
equilibrium allocation \( \{C_{h,t}, C_{f,t}, N_t, C_{h,t}, C_{f,t}, N_t^* \} \), the equilibrium conditions are all satisfied with constant producer price levels in each country equal to arbitrary numbers, and a constant nominal exchange rate.

The proposition follows:

**Proposition 1.** Let \( P_{h,0} \) and \( P_{f,0}^* \) be arbitrary positive numbers. Any flexible equilibrium allocation can be implemented with \( P_{h,t} = P_{h,0} \) and \( P_{f,t}^* = P_{f,0}^* \), and constant exchange rates, \( \epsilon_t = \epsilon_0 \).

**Proof.** Without loss of generality we take \( T = 1 \). In the beginning of period \( t = 2 \) the assets market opens to liquidate debts. This means that the wealth of the households in period \( t = 2 \), in either country, is zero, \( W_2 = 0 \) and \( W_2^* = 0 \).

We take as given an arbitrary equilibrium allocation \( \{C_{h,t}, C_{f,t}, N_t, C_{h,t}, C_{f,t}, N_t^* \} \) for \( t = 0, 1 \), in the set defined above. We show that there are constant prices with \( P_{h,t} = P_{h,0} \), \( P_{f,t}^* = P_{f,0}^* \), and fixed exchange rates, \( \epsilon_t = \epsilon_0 \), which implies that \( R_t = R_t^* \), that satisfy the equilibrium equations for that allocation which are \( (6)-(19), (22)-(29) \) and \( (31) \).

First, this allocation satisfies trivially the two feasibility constraints, \( (24) \) and \( (25) \), as it is an equilibrium allocation. For given \( P_{h,0} \), \( P_{f,0}^* \), we use the remaining equilibrium conditions to determine the values for the policy variables and remaining prices.

The firms’ conditions
\[
\begin{align*}
W_t &= \frac{\theta - 1}{\theta} A_t, \quad t = 0, 1, \\
W_t^* &= \frac{\theta - 1}{\theta} A_t^*, \quad t = 0, 1,
\end{align*}
\]
determine \( W_t \) and \( W_t^* \). The period 0 intertemporal budget constraints for the two representative households are
\[
\begin{align*}
\sum_{t=0}^{1} \beta^t E_0 \left[ (u_{C_h}(t)C_{h,t} + u_{C_f}(t)C_{f,t} - u_L(t)N_t) \right] &= \mathbb{W}_0 \frac{u_{C_h}(0)}{P_{h,0}(1 + \tau_{h,0})}, \\
\sum_{t=0}^{1} \beta^t E_0 \left[ (u_{C_h}^*(t)C_{h,t}^* + u_{C_f}^*(t)C_{f,t}^* - u_L^*(t)N_t^*) \right] &= \mathbb{W}_0^* \frac{u_{C_h}^*(0)}{P_{h,0}(1 + \tau_{h,0}^*)},
\end{align*}
\]
which are satisfied by appropriately choosing \( \tau_{h,0} \) and \( \tau_{h,0}^* \). Given a common process for the nominal interest rate
\[
R_t = R_t^*
\]
to be determined later, and \( \tau_{h,0} \) and \( \tau_{h,0}^* \), we can use
\[
\begin{align*}
\frac{u_{C_h}(0)}{(1 + \tau_{h,0})} &= \beta R_0 E_0 \frac{u_{C_h}(1)}{(1 + \tau_{h,1})}, \quad (32) \\
\frac{u_{C_h}^*(0)}{(1 + \tau_{h,0})} &= \beta R_0 E_0 \frac{u_{C_h}^*(1)}{(1 + \tau_{h,1})}, \quad (33)
\end{align*}
\]
\[
u_{C_h}(1)C_{h,1} + u_{C_f}(1)C_{f,1} - u_L(1)N_1 = \mathbb{W}_1 \frac{u_{C_h}(1)}{P_{h,0}(1 + \tau_{h,1})}, \quad s^1 \in S^1, \quad (34)
\]
\[ u_{C_h}^*(1)C_{h,1}^* + u_{C_f}^*(1)C_{f,1}^* - u_{L,t}^*(1)N_{t}^* = W_{1,0}^* \frac{u_{C_h}^*(1)}{\varepsilon_{0}^*(1 + \tau_{h,t}^*)}, \quad s^1 \in S^1, \]  
(35)
to determine the levels of noncontingent assets, \(W_1\) and \(W_1^*\), and contingent taxes, \(\tau_{h,t}, \tau_{h,t}^*\).

The marginal conditions for the state-contingent assets, (8) and (9), are satisfied by the choice of \(Q_{t-1,t}\) and \(Q_{t-1,t}^*\), \(t = 1\). The intratemporal conditions

\[
\begin{align*}
\frac{u_L(t)}{u_{C_h}(t)} &= \frac{W_t(1 - \tau_{n,t})}{P_{h,0}R_t(1 + \tau_{h,t})}, \quad t = 0, 1, \\
\frac{u_{C_h}(t)}{u_{C_f}(t)} &= \frac{(1 + \tau_{h,t})P_{h,0}}{(1 + \tau_{f,t})\varepsilon_{0}P_{f,0}}, \quad t = 0, 1, \\
\frac{u_{L,t}^*}{u_{C_f}(t)} &= \frac{W_t^*(1 - \tau_{n,t}^*)}{P_{f,0}R_t(1 + \tau_{f,t}^*)}, \quad t = 0, 1, \\
\frac{u_{C_h}^*(t)}{u_{C_f}^*(t)} &= \frac{(1 + \tau_{h,t}^*)P_{h,0}}{(1 + \tau_{f,t}^*)\varepsilon_{0}P_{f,0}}, \quad t = 0, 1
\end{align*}
\]

are satisfied by the choice of tax rates, \(\tau_{n,t}, \tau_{f,t}, \tau_{h,t}^*, \tau_{f,t}^*\), for \(t = 0, 1\). The cash-in-advance constraints

\[
(1 + \tau_{h,t})P_{h,0}C_{h,t} + (1 + \tau_{f,t})\varepsilon_{0}P_{f,0}^*C_{f,t} \leq M_t, \quad t = 0, 1, \\
(1 + \tau_{h,t}^*)P_{h,0}^*C_{h,t}^* + (1 + \tau_{f,t}^*)P_{f,0}^*C_{f,t}^* \leq M_t^*, \quad t = 0, 1
\]

are satisfied by the choice of \(M_t\) and \(M_t^*\), for \(t = 0, 1\).

The home country intertemporal budget constraints

\[
\begin{align*}
W_1^e &= \frac{1}{R_1} [\varepsilon_0 P_{f,0}^* C_{f,1} - P_{h,0}^* C_{h,1}^*], \quad s^1 \in S^1, \\
W_0^e &= \frac{1}{R_0} [\varepsilon_0 P_{f,0}^* C_{f,0} - P_{h,0}^* C_{h,0}^*] + E_0 \frac{Q_{0,1}}{R_1} \left[\varepsilon_0 P_{f,0}^* C_{f,1} - P_{h,0}^* C_{h,1}^*\right]
\end{align*}
\]

are satisfied by choices of the nominal interest rates in the two periods, \(R_1\) and \(R_0\), and the initial value of the nominal exchange rate, \(\varepsilon_0\). There are still degrees of freedom for the level of the nominal exchange rate, \(\varepsilon_0\), and the level of noncontingent total assets, \(W_1^e\).

The proof extends to any finite horizon economy, \(t = 0, \ldots, T\), with \(T\) arbitrarily large. \(\square\)

We have shown that for any equilibrium allocation, \(\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}\), the equilibrium conditions can be satisfied by asset positions, prices and policies such that producer prices are constant and equal to arbitrary constants, \(P_{h,t} = P_{h,0}, P_{f,t} = P_{f,0}^*\), and exchange rates are also constant, \(\varepsilon_t = \varepsilon_0\).

Taxes play a particular role when equilibria have constant producer prices and exchange rates. Since prices are constant and so is the exchange rate, consumption taxes in one good relative to the other have to move if relative prices are to move, which is the case in general if there are different shocks in different countries. Consumption taxes play another role, when nominal public debt is noncontingent, that of replicating state-contingent real debt. The price level gross of consumption taxes is the deflator of nominal debt, so that ex-post volatility of the consumption tax can replicate state-contingent real debt, which is the role played by ex-post volatility of the price level in Chari et al. [8]. We have assumed, as is standard in this literature, that internationally
traded assets are state-noncontingent. Nominal interest rates, that in a fixed exchange rate regime are common across countries, can play the role of replicating state-contingent international debt. Consumption taxes also affect the households margin between consumption and labor. Labor income taxes have to adjust to compensate those effects. Since prices are constant and technologies in the two countries are varying, the nominal wages have to move in response to shocks, and move differently in different countries. Nominal interest rates, that in a fixed exchange rate regime are common across countries, can play the role of replicating state-contingent international debt. Money supply also has to move in response to shocks, in order to satisfy the cash-in-advance constraints.

One first implication of the result in the proposition is that fixed exchange rates do not restrict the set of allocations under flexible prices. This is an interesting result in itself, in particular, since asset markets are incomplete in our model. However, the issue of whether there are costs of a fixed exchange rate regime is typically associated with the presence of some type of price rigidity, as first argued by Friedman [20]. If prices are sticky and exchange rates are fixed, one would think that there would be restrictions on the relative prices of the goods produced in the different countries. That is not the case: The second implication of the proposition is that fixed exchange rates do not restrict the set of allocations when prices are sticky. In the following section we assume that firms are restricted in the setting of prices.

4. Sticky prices

We assume that firms set prices as in Calvo [7] staggered price setting, which is a commonly used assumption in the sticky price literature. We assume that the firms set prices in the domestic currency. In each country, starting from an historical common price, at every date, each firm can optimally set its price with some probability, that can differ across countries. Because there is a continuum of firms, the probability is also the share of firms that optimally revise the price in each period.

In general, staggered price setting leads to inefficient differences in prices across firms. Although in a given country firms are otherwise identical, have the same linear technology and face identical demand functions, they may charge different prices. The only case in which this will not occur is when firms that are able to change prices decide not to do it. The price setting restrictions in this case will not be binding, and the producer price level in each country will be constant. The equilibrium conditions will be identical to the equilibrium conditions of the flexible price economy when producer prices are constant over time.

Since, as stated in Proposition 1, under flexible prices it is possible to implement the full set of equilibrium allocations with constant prices and fixed exchange rates, it follows that, under sticky prices, it is also possible to implement that same set, also with fixed exchange rates.

It is clear that under sticky prices there are allocations that are not implementable under flexible prices. That is the case whenever otherwise identical firms set different prices. It turns out, as we show in Appendix A, that the set of flexible price allocations dominates in terms of welfare the set of allocations under sticky prices. Since agents are heterogeneous across countries, the meaning of welfare dominance is the usual one, of a potential Pareto movement where lump-sum transfers between agents are implicitly assumed.

Independently of the exchange rate regime, for each allocation that can be implemented under sticky prices there is one under flexible prices that can potentially improve welfare in both coun-

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11 If wages were sticky, additional taxes would be needed to keep the wages received by the households constant, as well.

12 This would be the case in a steady state where the prices are constant over time and therefore the same across firms.
tries. The reason for this is that, in order for sticky prices to be relevant, because different firms face different price setting restrictions, firms that use the same technology and face the same demand conditions would charge different prices. This means that production would be inefficient and the inefficiency in production is not optimal in this second best environment.\footnote{This result, that replicating flexible prices is optimal, would be straightforward in a first best, if sticky prices were the sole distortion. Instead, in a second best where there are other distortions, adding an additional distortion could improve welfare.}

This result relates to the one in Diamond and Mirrlees [16] that shows that it is not optimal to tax intermediate goods when there are consumption taxes on the final goods.

The proposition follows.

**Proposition 2.** In a world economy with noncontingent bond markets and Calvo (1983) staggered price setting there is no cost of a fixed exchange rate regime, independently of the degree of price rigidity.

**Proof.** In Proposition 1 we show that the set of allocations under flexible prices is implemented with policies that generate constant prices, equal to arbitrary numbers, and constant exchange rates. For the policies that induce prices to be equal to the historical initial prices of the Calvo firms, \( P_{h,0} \) and \( P_{f,0}^* \), the equilibrium conditions under Calvo [7] are exactly the same ones as under flexible prices. This establishes that the flexible price set of allocations is implementable under Calvo price setting, with fixed exchange rates. It remains to show that the set is optimal, in the sense that for every allocation in the set under sticky prices, there is one in the set under flexible prices that is a potential Pareto improvement. This is done in Appendix A.

The result in this proposition can be extended to any other form of price stickiness, such as prices set in advance as in Ireland [22], Taylor [29] staggered prices, Rotemberg [26] adjustment costs of changing prices, or state-dependent pricing as in Dotsey, King and Wolman [17]. For the case where prices are set in advance, let the initial prices \( P_{h,0} \) and \( P_{f,0}^* \) be exogenously given and the other period prices, \( P_{h,t} \) and \( P_{f,t}^* \), be set in advance for \( k \) periods, for a finite \( k \). Proposition 1 implies that adding those restrictions to the flexible price economy still allows to implement the set of allocations under flexible prices, in a fixed exchange rate regime. The argument of welfare dominance of the flexible price set also applies there.

We have assumed that prices are set in the currency of the producer. We could alternatively have assumed local currency pricing. The results would follow through. For the policies that support constant producer prices and constant exchange rates, local currency pricing would not have any impact. Contrary to what is argued extensively in the literature that does not allow for fiscal policy instruments, it does not make a difference whether prices are set in the currency of the producer or the consumer.

We have analyzed flexible versus fixed exchange rate regimes. The analysis clearly follows through in a monetary union.

5. Minimal set of instruments

In Appendix A, in order to show the optimality of flexible prices over sticky prices, we characterize the set of equilibrium allocations that can be implemented in the model of Sections 2 and 3. The model is without capital, without mobility of labor or state-contingent assets, with flexible
prices and wages, and flexible exchange rates. The fiscal instruments are labor income taxes, taxes on the consumption of home and foreign goods in each country, and public debt is non-contingent. Because we are interested in the Pareto frontier, we assume that there are lump-sum transfers across countries. The set of implementable allocations is characterized by two implementability conditions,\textsuperscript{14} corresponding to the time zero budget constraints of the governments (or households) in each country – in the primal form, in terms of quantities – (42) and (43), and the resource constraints for each of the two goods in each state, (44) and (45).

This implementable set looks like the one in Lucas and Stokey \[23\] for a closed economy, without capital, with flexible prices and wages. In the closed economy of Lucas and Stokey, with only one consumption good, the implementability conditions are the single implementability condition corresponding to the time 0 budget constraint of the single government, and the resource constraints for the single good, for each state. Lucas and Stokey \[23\] consider different fiscal instruments. The taxes are labor income taxes only, and public debt is state-contingent.

What is the minimal set of instruments to implement the equilibrium allocations characterized by those implementability and feasibility conditions in Appendix A, analogous to the ones in Lucas and Stokey? How important are assumptions about the environment, flexible prices, fixed exchange rates versus flexible, labor mobility and mobility of assets, capital accumulation, for that minimal set of instruments?

In the basic setup without mobility of labor or of state-contingent private assets, and with noncontingent public debt, but with all prices flexible, including exchange rates and wages, the minimal set of instruments are labor income taxes, and one additional tax on one of the consumption goods in one of the countries, a total of three taxes for the two countries. The need for this one additional tax is that in this model there are two agents and two goods. Without that consumption tax, the marginal rates of substitution between the two consumption goods would have to be equal for the two agents, and that would be an additional implementability constraint.

Whether public debt is contingent or not is actually irrelevant for the implementable set since it is possible to replicate state-contingent debt with ex-post volatility of the price level, a point made in the closed economy by Chari et al. \[8\].

In this flexible price model, if the exchange rate was fixed, then the same set of equilibrium allocations could be implemented with one consumption tax in each country, so that there would be then a total of four taxes, on labor income in each country and on one consumption good in each country. This is a case where one instrument is replaced by another equivalent instrument.

The same implementable set, considered in Appendix A, can be achieved even if prices are sticky, in a fixed exchange rate regime, without state-contingent public debt, provided other instruments are used (Proposition 1). In order to neutralize the sticky price restrictions, the equilibria must be implemented with constant prices. Because of those additional constraints on prices, we need to use additional fiscal instruments. We assume that, in addition to labor income taxes, there are consumption taxes on all goods, rather than in one good only.\textsuperscript{15}

Consumption taxes on one good, possibly the domestic good, in each country have to play the role of price level volatility in replicating state-contingent public debt. They also play the role of equating nominal interest rates to the real rates plus inflation gross of taxes. With fixed exchange rates, there is a need for consumption taxes in the two other consumption goods to implement

\textsuperscript{14} The lump-sum transfers are important so that there is not a third constraint, the intertemporal budget constraint of the country.

\textsuperscript{15} If wages were sticky, in order to neutralize those restrictions, wages would have to be constant in equilibrium. This could be achieved with additional tax instruments, such as payroll taxes.
the desired intratemporal margins across consumption goods. These are minimal instruments to attain the irrelevance results.

Adding further decisions by private agents, such as the choice of working in a different country or holding assets in different currencies, or capital accumulation, requires further instruments. Without further instruments, a fixed exchange rate regime would not be costless. We now consider those three cases.

5.1. Labor mobility and completeness of financial markets

We have assumed that labor is not mobile across countries, and that financial markets are to some extent segmented, since state-contingent debt can only be traded within the country. We do not think that labor mobility across countries at the business cycle frequency is in any way relevant. Still, the question of what would be the implications of labor mobility is of theoretical interest, specially because we obtain a very different result from the one in the literature. That is the case, as well, for mobility of financial assets.

We show that absence of labor mobility is a necessary condition for the irrelevance results. We also show that if private state-contingent debt was traded internationally a similar result would be obtained. In either case, in order to obtain the irrelevance results, it would be necessary to have further instruments. These could be differential taxes depending on where the income originates from, in the case of labor mobility, and state-contingent public debt for the case of capital mobility.

5.1.1. Labor mobility

We assume, now, that workers can choose to work in foreign firms being taxed at home. They consume at home. This is one way of modeling labor mobility. There are alternative ways, but the same arguments go through.

For the home households, total labor \( N_t \) is split between work at home \( N_{h,t} \) and work abroad \( N_{f,t} \), \( N_t = N_{h,t} + N_{f,t} \). Similarly, for the foreign country, \( N_t^* \) is split between \( N_{h,t}^* \), which is labor abroad, and \( N_{f,t}^* \), labor at home, in the foreign country, \( N_t^* = N_{h,t}^* + N_{f,t}^* \). The market clearing conditions in the goods market, (24) and (25), become

\[
C_{h,t} + C_{h,t}^* + G_t = A_t \left[ N_{h,t} + N_{h,t}^* \right]
\]

and

\[
C_{f,t} + C_{f,t}^* + G_t^* = A_t^* \left[ N_{f,t} + N_{f,t}^* \right].
\]

The conditions of the households problems are the same except for an additional arbitrage condition on where to work, that equates the two wages,

\[
W_t = \varepsilon_t W_t^*.
\]  

(38)

The price setting conditions for the firms are also unchanged. The home country budget constraints (31) become

\[
\sum_{s=t}^{T} E_t Q_{t,s+1} \left[ \varepsilon_s P_{f,s} C_{f,s} - P_{h,s} C_{h,s} + W_s N_{h,s}^* - \varepsilon_s W_s^* N_{f,s} \right] = \mathbb{W}_t - \mathbb{W}_t^*, \quad \text{all } s', 0 \leq t \leq T.
\]

Notice that full labor mobility implies one additional constraint per state to the equilibrium conditions, namely (38). The wage in the same currency must be equal across countries. This is
not possible to satisfy with fixed exchange rates. In particular, it is not possible to satisfy both the
firms price setting conditions (22) and (23), with \( P_{h,t} = P_{h,0} \) and \( P_{f,t}^* = P_{f,0}^* \), and the arbitrage
condition (38) with \( \varepsilon_t = \varepsilon_0 \).

It follows that, if labor was mobile, the policy instruments considered above would not be
sufficient to implement the set of flexible equilibrium allocations with fixed exchange rates. Ad-
ditional fiscal instruments would be needed. If it was possible to tax labor of migrants differently,
then there would be again no costs of the exchange rate regime, with labor mobility.

In the Mundellian optimal currency area literature, labor mobility made the costs of a currency
area lower. Labor mobility reduced the costs of Keynesian unemployment and, therefore, the
need for stabilization policy. We obtain the opposite result, for a very different reason. Labor
mobility makes it harder to establish the result that there are no costs of a monetary union, in
the sense that it would require more policy instruments. In our set up, more flexibility in private
decisions makes the life of the planner harder.

The fact that with labor mobility there are costs of a fixed exchange rate regime, while there
are no such costs when labor is immobile, does not mean that labor mobility is undesirable. We
are not comparing environments with and without labor mobility, but rather environments with
and without fixed exchange rates, when labor is immobile or when it is mobile.

5.1.2. Completeness of financial markets

We have assumed that public debt was noncontingent and that state-contingent private debt
was traded only within the country. If public debt was state-contingent, the results would not
be affected. It would mean that there would be more instruments and, therefore, if we obtained
the results with less instruments we would also obtain them with more. Since the private agents
already had access to domestic state-contingent nominal assets, there would be no change in
marginal private decisions. The only difference in the equilibrium conditions would be that the
budget constraints (18) and (19) could then be satisfied with the choice of \( \mathbb{W}_t \) and \( \mathbb{W}_t^* \) because
these would be state-contingent.

The interesting cases are the following two cases, (a) and (b): In both private state-contingent
debt was traded internationally. In (a) public debt is state-contingent, while, in (b), public debt is
not state-contingent.

In (b), the irrelevance results would not go through. The additional private decisions would
add equilibrium conditions that could not be satisfied with the existing instruments, with fixed
prices and exchange rates. This result is analogous to the one on labor mobility. Instead in (a),
the additional instruments provided by state-contingent public debt are enough to satisfy those
additional arbitrage conditions. We show this now.

Private state-contingent debt is traded internationally, and public debt is state-contingent. If
private state-contingent debt can be traded internationally, then there are additional marginal
conditions. The agents can arbitrage between the two nominal state-contingent assets, in the two
currencies, so that the following conditions must be satisfied:

\[
Q_{t-1,t}^* = \frac{Q_{t-1,t}^* \varepsilon_{t-1}}{\varepsilon_t}, \quad \text{all } s^{t-1} \text{ and } s^t | s^{t-1}, \quad 1 \leq t \leq T. \tag{39}
\]

With a constant exchange rate, these restrictions imply

\[
\frac{u_{C_h}(t-1) + \tau_{h,t}}{\beta u_{C_h}(t)} = \frac{u_{C_h}(t-1) + \tau_{h,t-1}^*}{\beta u_{C_h}(t) + \tau_{h,t}}, \quad \text{all } s^{t-1} \text{ and } s^t | s^{t-1}, \quad 1 \leq t \leq T. \tag{39}
\]
It turns out that the supply of state-contingent public debt frees up enough instruments that can be used to satisfy these conditions.

As mentioned above, because public debt is state-contingent, the budget constraints in period 1 in the two countries, \( (34) \) and \( (35) \), are now satisfied by the choice of \( \mathbb{W}_1 \) and \( \mathbb{W}_1^* \), so that the consumption taxes, \( \tau_{h,1} \) and \( \tau_{h,1}^* \), are not restricted by these conditions. Because the state-contingent assets are traded internationally, the home country budget constraints in period 1, \( (36) \), can be satisfied by the choice of \( \mathbb{W}_1^* \), so that the nominal interest rates, \( R_1 \), are free. The country budget constraint for period \( t = 0 \), \( (37) \), determines \( \epsilon_0 \), so that \( R_0 \) is also free.

Given \( \tau_{h,0} \) and \( \tau_{h,0}^* \), the intertemporal marginal conditions \( (32) \) and \( (33) \) are satisfied by the choice of \( \tau_{h,1} \). The tax rates \( \tau_{h,1} \) are not restricted and, therefore can be used to satisfy the arbitrage restrictions \( (39) \), above.

**Private state-contingent debt is traded internationally and public debt is not state-contingent.** If state-contingent private debt could be traded internationally, but public debt was not state-contingent, then it would not be possible to satisfy the arbitrage conditions \( (39) \). The consumption taxes would be used to replicate state-contingent public debt and to satisfy the intertemporal conditions \( (32) \) and \( (33) \), and there would not be enough degrees of freedom to satisfy the arbitrage conditions \( (39) \).

This result is analogous to the result on labor mobility. Also here, if financial markets were integrated, in the sense that private state-contingent assets could be traded internationally, then there would be costs of a fixed exchange rate regime. Unless, again, additional instruments would be considered, such as state-contingent public debt, as shown in the case above.

### 5.2. Capital accumulation

With capital as an additional input, the production functions in the home and foreign country are, \( Y_{h,t}(i) = A_t F(K_{t-1}^*(i), N_t(i)) \) and \( Y_{f,t}(j) = A_t^* F(K_{t-1}^*(i), N_{t}^*(j)) \), respectively. We assume, for simplicity, that capital is produced at home, so that the market clearing conditions, in the two countries, are \( C_{h,t} + C_{h,t}^* + G_t + K_t - (1 - \delta)K_{t-1} = A_t F(K_{t-1}, N_t), \) all \( s' \), \( 0 \leq t \leq T \), and \( C_{f,t} + C_{f,t}^* + G_t^* + K_t^* - (1 - \delta)K_{t-1}^* = A_t^* F(K_{t-1}^*, N_t^*), \) all \( s' \), \( 0 \leq t \leq T \), where \( \delta \) is the depreciation rate.

The flexible-price firm decisions, in each country, include the choice of the quantity of capital given the rental price of capital, \( U_t \) and \( U_t^* \). Those decisions add the same number of equilibrium conditions as unknowns.

The households hold capital. Their arbitrage conditions are

\[
R_t P_{h,t} = E_t \left[ P_{h,t+1} + (1 - \tau_{k,t+1})(U_{t+1} - \delta P_{h,t+1}) \right], \quad \text{all } s', \ 0 \leq t \leq T - 1, \quad (40)
\]

\[
R_t^* P_{f,t}^* = E_t \left[ P_{f,t+1}^* + (1 - \tau_{k,t+1}^*)(U_{t+1}^* - \delta P_{f,t+1}^*) \right], \quad \text{all } s', \ 0 \leq t \leq T - 1, \quad (41)
\]

where \( \tau_{k,t+1} \) and \( \tau_{k,t+1}^* \) are the capital income taxes in the two countries. In order to satisfy these conditions when the interest rates are equal, \( R_t = R_t^* \), and the prices are constant, it is necessary to use further instruments. The capital income taxes play that role.

### 6. Concluding remarks

In this paper we address the central issues in the literature on optimal currency areas using the approach of Ramsey optimal fiscal and monetary policy. In our set up, every currency area
is an optimal currency area, provided there are restrictions on the mobility of labor and financial assets. This extreme result is in sharp contrast with the literature on optimal currency areas, and calls for the need to take into account fiscal policy when addressing those regime issues.

Under a flexible exchange rate regime, monetary policy in each country can freely respond to shocks; it may respond to country specific shocks or it may respond to common shocks in different ways. Instead, in a monetary union there is a unique monetary policy for the members of the union. This implies restrictions in the use of policy; the exchange rate must be constant over time and the nominal interest rate must be equal across countries. Are these restrictions relevant to achieve the optimal equilibrium allocations? Does the answer to this question change with the introduction of nominal rigidities, like staggered price setting? Does it matter that international capital markets are segmented, and that labor is not mobile?

The conventional, Mundellian, wisdom is that there are costs of a fixed exchange rate regime, or a monetary union, resulting from the loss in ability to use policy for stabilization purposes. The costs are taken to be higher the stronger are the asymmetries across countries in shocks and their transmission, and the stronger are the nominal rigidities. Instead, we show that in an environment with nominal rigidities, whatever the type of price setting, producer currency pricing or local currency pricing, the exchange rate regime, whether flexible or fixed exchange rates, is irrelevant once fiscal policy instruments are taken into account. We also show that, in order for the costs of the monetary union to be zero, labor cannot be mobile, unless additional policy instruments were used. Similarly, there must be restrictions on the mobility of financial assets.

One final comment: Friedman [20] made the point that, in a world with sticky prices, exchange rates should be flexible in order for relative prices to be adjusted. We make a further point that in a world where prices are sticky and exchange rates cannot move there are still policy instruments that can replace the role of the price level and the exchange rate. Note that the adjustments in fiscal policy are not automatic and would require a knowledge of the model and the shocks to be fully effective. But the movements in exchange rates that would be necessary to accomplish the same goal, could be market determined, but would not be automatic either. The information that is necessary to conduct policy under flexible exchange rates so that the path for exchange rates is a particular one is exactly the same information necessary to affect directly the relative prices using tax rates.

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Appendix A. Allocations under flexible and sticky prices

In this appendix we show that for each allocation under sticky prices, there is an allocation under flexible prices that gives at least as high welfare to one country without reducing the welfare of the other country.

Assuming that lump-sum transfers are feasible between countries, the set of implementable allocations under flexible prices \( \{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\} \) as well as initial taxes, prices, and exchange rate \( \{\tau_{h,0}, \tau_{h,0}^*, P_{h,0}, \varepsilon_0\} \) is characterized by the following conditions:
\[ \sum_{t=0}^{1} \beta^t E_0 \left[ (u_{Ch}(t) C_{h,t} + u_{Cf}(t) C_{f,t} - u_{L}(t) N_t) \right] = \mathbb{W}_0 \frac{u_{Ch}(0)}{P_{h,0}(1 + \tau_{h,0})}, \]  
(42)

\[ \sum_{t=0}^{1} \beta^t E_0 \left[ (u_{Ch}^*(t) C_{h,t}^* + u_{Cf}^*(t) C_{f,t}^* - u_{L}^*(t) N_t^*) \right] = \mathbb{W}_0^* \frac{u_{Ch}^*(0)}{P_{h,0}^*(1 + \tau_{h,0}^*)}, \]  
(43)

\[ C_{h,t} + C_{h,t}^* + G_t = A_t N_t, \]  
(44)

\[ C_{f,t} + C_{f,t}^* + G_t^* = A_t^* N_t^*. \]  
(45)

We do not impose as a restriction the budget constraint between countries, because we allow for transfers between these. The remaining equilibrium conditions determine the policy and prices. Denote the set of allocations that satisfy these conditions by \( E_f \).

Under sticky prices the set of equilibrium conditions cannot be summarized by a small set of implementability conditions as under flexible prices. The allocations \( \{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\} \) are restricted by the same intertemporal budget constraints as in the flexible price case above, (42) and (43). For given prices \( \{\frac{P_{h,t}(i)}{P_{h,t}}, \frac{P_{f,t}(j)}{P_{f,t}}\} \), they are restricted by the feasibility conditions

\[ (C_{h,t} + C_{h,t}^* + G_t) \int_0^1 \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} di = A_t N_t, \]  
(46)

\[ (C_{f,t} + C_{f,t}^* + G_t^*) \int_0^1 \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\theta} dj = A_t^* N_t^*, \]  
(47)

the conditions that define the aggregate price levels, \( P_{h,t} = [\int_0^1 P_{h,t}(i)^{1-\theta} di]^{\frac{1}{\theta}} \) and \( P_{f,t}^* = [\int_0^1 P_{f,t}(j)^{1-\theta} dj]^{\frac{1}{\theta}} \), as well as all the remaining equilibrium conditions. Let the set of allocations that satisfy these restrictions be denoted by \( E^s \).

It is straightforward to show that \( D = \int_0^1 \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} di \geq 1 \) and \( D^* = \int_0^1 \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\theta} dj \geq 1 \).

\( D = 1 \) when \( \frac{P_{h,t}(i)}{P_{h,0}} = 1 \), and \( D^* = 1 \) when \( \frac{P_{f,t}(j)}{P_{f,0}} = 1 \).

The set of allocations under flexible prices dominates the set under sticky prices, meaning that for each allocation in \( E^s \) there is at least one allocation in \( E_f \) with at least one of the goods in larger or equal quantity and none smaller. The intertemporal budget constraints are the same but the feasibility conditions are different, being (46) and (47) more restrictive than (44) and (45), and there are additional equilibrium restrictions over \( E^s \) that are absent from \( E_f \). Moreover, the restrictions over the allocations under sticky prices are exactly the same only when \( P_{h,t}(i) = P_{h,0} \) and \( P_{f,t}(j) = P_{f,0} \).

References