MICROECONOMICS II
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EXERCISES (Answers)
PART II

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4. Duality in Producer Theory

Exercise 1

a)

\[ y_1 \rightarrow IRS \quad \text{se} \quad a + b > 1 \]
\[ y_1 \rightarrow CRS \quad \text{se} \quad a + b = 1 \]
\[ y_1 \rightarrow DRS \quad \text{se} \quad a + b < 1 \]
\[ y_2 \rightarrow IRS \quad \text{se} \quad h > 1 \]
\[ y_2 \rightarrow CRS \quad \text{se} \quad h = 1 \]
\[ y_2 \rightarrow DRS \quad \text{se} \quad h < 1 \]

\[ y_1 \rightarrow CRS \]

b) \( \varepsilon_{y_t,K} = a > 0 \quad \varepsilon_{y_t,L} = b > 0 \)

\[ \varepsilon_{y_t,K} = \frac{hd}{d + (1-d) \left( \frac{L}{K} \right)} > 0 \quad \varepsilon_{y_t,L} = \frac{h(1-d)}{(1-d) + d \left( \frac{K}{L} \right)} > 0 \]

\[ \varepsilon_{y_t,K} = \begin{cases} 1 & K < (a/b) \cdot L \\ 0 & K \geq (a/b) \cdot L \end{cases} \]
\[ \varepsilon_{y_t,L} = \begin{cases} 1 & L < (b/a) \cdot K \\ 0 & L \geq (b/a) \cdot K \end{cases} \]

c) \( \sigma_1 = 1 \)
\[ \sigma_2 = \frac{1}{e + 1} \]
\[ \sigma_3 = 0 \]

Exercise 2

a) -

b) IRS

c) \( Y_2 = KL \)

Exercise 3

a) Hotelling’s Lemma: \( y(p, w_1, w_2) = \frac{P}{2} \left( w_1^{\frac{2}{3}} + w_2^{\frac{2}{3}} \right)^\frac{3}{2} \)

\[ \frac{\partial y}{\partial p} > 0; \quad \frac{\partial y}{\partial w_1} < 0; \quad \frac{\partial y}{\partial w_2} < 0 \]

b) The supply function is homogeneous of degree 0.
Exercise 4

a) \( \text{Min} C = wL + rK \) s.a. \( \bar{y} = K^a L^b \)

b) \( L(r, w, y) = \frac{1}{y^{a+b}} \cdot \left( \frac{r}{w} \cdot \frac{b}{a} \right)^{\frac{a}{a+b}} \)

\( K(r, w, y) = \frac{1}{y^{a+b}} \cdot \left( \frac{w}{r} \cdot \frac{a}{b} \right)^{\frac{b}{a+b}} \)

c) \( C(r, w, y) = w \cdot \frac{1}{y^{a+b}} \cdot \left( \frac{r}{w} \cdot \frac{b}{a} \right)^{\frac{a}{a+b}} + r \cdot \frac{1}{y^{a+b}} \cdot \left( \frac{w}{r} \cdot \frac{a}{b} \right)^{\frac{b}{a+b}} \)

d) \( \text{Max} \Pi = p \cdot K^a L^b - wL - rK \)

e) \( L(r, w, p) = \left( \frac{w}{b} \right)^{\frac{1-a}{a+b-1}} \cdot \left( r \right)^{\frac{a}{a+b-1}} \cdot p \left( \frac{1}{a+b-1} \right) \)

\( K(r, w, p) = \left( \frac{w}{b} \right)^{\frac{b}{a+b-1}} \cdot \left( r \right)^{\frac{1-b}{a+b-1}} \cdot p \left( \frac{1}{a+b-1} \right) \)

f) \( \Pi = p \left( \frac{1}{a+b-1} \right) \left[ \left( \frac{w}{b} \right)^{\frac{b}{a+b-1}} \cdot \left( r \right)^{\frac{a}{a+b-1}} - w \cdot \frac{1-a}{a+b-1} \right] \) - wL(w, r, p) - r \cdot K(w, r, p)

Two different strategies:
- \( \Pi = p \cdot K(w, r, p) - L(w, r, p) - wL(w, r, p) - r \cdot K(w, r, p) \)
- \( \Pi = p \cdot y(w, r, p) - C[w, r, y(w, r, p)] \)
5. Input Markets

Exercise 1
a) \( x = 9; Q = 6; \Pi = 9 \)
b) \( x = 4; p = 4; Q = 4; \Pi = 12 \)
c) \( x = 6; Q = 4.899; w = 0.8165; \Pi = 9.7979 \)

Exercise 2
a) \( L = 9; Q = 6; \Pi = 31 \)
b) \( L = 4; Q = 4; p = 12; \Pi = 27 \)
c) \( L = 4; Q = 4; w = 4; \Pi = 27 \)
d) \( L = 3.09489; w = 3.54745; Q = 3.51846; p = 12.48154; \Pi = 27.93683 \)

Exercise 3
a) \( Y = \frac{25}{(1 + P_y)^2} \)
b) \( P_x = 7.5; P_y = 1; X = 2.5; Y = 6.25; Z = 12.5 \)
c) Firm A would be willing to pay 2.5 \( (= \Pi_B) \).
   With DRS, Firm A would be willing to pay more.

Exercise 4
Union B.

Exercise 5
a) \( L = Y = \frac{25 - w}{2} \)
b) \( w = 13; L = Y = 6; p = 19; \Pi_y = 36; \text{Surplus} = 72 \)
c) Without the association:
   \( w = 1; L = Y = 12; p = 13; \Pi_y = 144; \text{Surplus} = 0 \)
d) \( \text{Surplus} \geq 72 \)

Exercise 6
a) \( X_i = \left( \frac{P_y - 5}{2P_x} \right)^2 \)
b) \( P_x = 104 \)
c) $P_y = 71,2745$

d) $X = 10,1523 ; Y = 31,8628$

e) $\Pi_i = 1,19$

f) Active Restriction $\rightarrow$ The monopolist’s profit is lower.

$Q \uparrow$ and $P \downarrow$ $\rightarrow$ The consumers’ surplus is higher.

**Exercise 7**

a) $p = 77,5; R = 47; X = Y = 22,5$

b) $p = 55; X = 45; \Pi = 2025$

**Exercise 8**

a) $P_x = 25; P_y = 2250; x = 10000; y = 2500$

b) Perfectly elastic supply.

**Exercise 9**

a) $P_x = 25; P_y = 400; x = 100; y = 25$

b) $P_x = 36; P_y = 390; x = 120; y = 30$

c) $P_x = 75; P_y = 400; x^D = 100; x^S = 173,205; y = 25$

**Exercise 10**

a) $L^D = 1200 - 40w$

b) $L = 600; \Pi = 9000$

c) $L = 720; \Pi = 6480$

d) $L = 850; \Pi = 7225; X = 16150; w = 10,5$

**Exercise 11**

a) $w = 2,25; L = 225$

$w = 2,5; L = 250$

b) 

i) $L = 300$

ii) $L = 225$

iii) $L = 180$

**Exercise 12**
a) \( L_A = 9; L_B = 10 \)

b) 

i. \( L_A = 8.5 \)

ii. \( w_B = 1.25 \)

Exercise 13

a) \( w_w = w_m = 180; L_w = 2000; L_m = 4000; X = 144000 \)

b) \( w_w = 100; w_m = 150; L_w = 1200; L_m = 2800; X = 104000 \)

c) \( w_w = w_m = 140; L_w = 1600; L_m = 2400; X = 104000 \)

Exercise 14

a) \( w_w = w_m = 25; L_w = 1000; L_m = 250; Q = 31250; \Pi = 0 \)

b) \( w_w = 12.5; w_m = 12.5; L_w = 500; L_m = 125; Q = 13625; \Pi = 7812.5 \)

c) \( w_w = 25; w_m = 1; L_w = 1000; L_m = 10; Q = 250; \Pi = 0 \)

6. General Equilibrium

Exercise 1

False.

Exercise 2

False.

Exercise 3

True, as long as both agents consume positive quantities and the MRS is well defined.

Exercise 5

a) \( p = \frac{\overline{Y}}{X} \)

b) Yes.

c) \( Y_A = \frac{\overline{Y}}{X} \cdot X_A \)

Exercise 6
\[ p = 0.05; X_{OC} = 20; X_{OR} = 30; Y_{OC} = 400; Y_{OR} = 200 \]

**Exercise 7**

a) A buys X and sells Y. B buys Y and sells X.
\[ \frac{1}{2} < p < 3 \]

b) Not a competitive equilibrium.

c) \( p = 1.214; X_A = 15.88; X_B = 14.12; Y_A = 12.86; Y_B = 17.14 \)

d) i) Leontief Preferences.

ii) \( Y_A = \frac{1}{2} X_A \)

iii) False.

**Exercise 8**

a) \( X = 2.828; O = 16; L = 8; \frac{\prod}{p} = 1.414 \)

b) \( X = 12; O = 12; L = 12; \frac{w}{\prod} = 1 \rightarrow \prod = 0 \)

**Exercise 9**

a) \( X = L_X = 12 \)

b) \( X = L_X = 6; Y = 2; L_Y = 4 \)

**Exercise 10**

a) \( X = 2; Y = 5 \)

b) Not a general equilibrium.

c) Lower.

**Exercise 11**

True.

**Exercise 12**

False.

**Exercise 13**

False.
Exercise 14

a) The contract curve is the diagonal of the Edgeworth Box.
The transformation curve has a constant slope of –1.
\[ p = 1 \]

b) \( \frac{X}{Y} = \frac{1}{4} \)

c) \( \frac{X}{Y} = \frac{1}{2} \)

However, the equilibrium relative price of inputs is the same.

Exercise 15

a) \( X = 190; Y = 760 \)

b) \( Y = 948.68 \)

c) \( X = 189.74; Y = 758.95 \)

Exercise 16

a) \( L_Y = 10K_Y \)

If \( X \uparrow \), then \( \frac{w}{r} \downarrow \).

b) -

c) \( L_X = 30; L_Y = 20; K_X = 8; K_Y = 2 \)

Exercise 17

a) Not a Pareto optimum.

b) \( p = 2; H_A = 15; S_A = 60; H_B = 45; S_B = 90 \)

c)  
  i. Not a Pareto improvement.
  ii. Equity.
  iii. Yes.

d)  
  i. \( p = TMT \)
  ii. -

Exercise 18

a) Depends if \( \alpha_1 \geq, < \) or \( \leq \alpha_2 \).
b) \( A_i = \frac{5w_i}{20-w_i} \)

c) \( w_i = \frac{5A_i}{1.5 \cdot A_i - 5} \)

\[ p = 1; A_1 = 5; A_2 = 10; w_1 = 10; w_2 = 5 \]

d) \( p \uparrow \)

There is a contraction of the PPF.

Exercise 19

a) \( K_X = L_X \)

b) \( X = L_X = K_X = 96; Y = 10.77; L_Y = K_Y = 4 \)

7. Welfare Economics

Exercise 1

Not necessarily true.

Exercise 2

False.

Exercise 3

a) The contract curve is the sides of the Edgeworth Box.

b) The fair allocations are those in which one of the two individuals has the whole endowment of one good and the other individual has the whole endowment of the other good.

Exercise 4

a) False.

b) False.

c) True.

Exercise 5

a) False.

b) If all individuals have the same Utility function and the weights of all Utility functions are the same.

Exercise 6

a) True.

b) Yes.

c) No.
Exercise 7
a) No decision.
b) Z will be chosen.
c) EDP can change the ordering to something different from its preferences in order to have Y chosen instead of Z.

Exercise 8
a) No.
b) Yes.