MICROECONOMICS II
2nd semester 2006/2007

EXERCISES
PART II

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4. Duality in Producer Theory

Exercise 1
Consider the technologies described by the following production functions:

\[ Y_1 = AK^a L^b \]
\[ Y_2 = A \left[ dK^{-c} + (1 - d)L^{-c} \right]^{-\frac{1}{c}} \]
\[ Y_3 = \min \left\{ \frac{K}{a}, \frac{L}{b} \right\} \]

a) Classify the technologies with respect to their returns to scale.
b) Find the output elasticities with respect to K and L in each case. Interpret.
c) Find the elasticities of substitution.

Exercise 2
The cost function of a certain firm is given by

\[ C(w_1, w_2, Y) = 2w_1 w_2 Y. \]

a) Check the properties of the cost function.
b) Analyse the existence of economies and diseconomies of scale. What can you conclude?
c) Consider the following production functions:

\[ Y_1 = K^{0.5} L^{0.5}, \quad Y_2 = KL, \quad Y_3 = K^{0.4} L^{0.6} \]

From which of them was the cost function constructed?

Exercise 3
You are an economic analyst at the firm Hotels and Company. From the scarce historical data available, the long-run profit function is given by

\[ \Pi(w_1, w_2, p) = \frac{p^2}{4} \left( w_1 \gamma_1 + w_2 \gamma_2 \right)^{\frac{3}{2}} \]

where \( w_1 \) and \( w_2 \) are the prices of the factors \( x_1 \) and \( x_2 \), and \( p \) the price of a hotel room.

a) Find the firm’s long-run supply function. Analyze and interpret the sign of the derivatives with respect to \( p \), \( w_1 \) and \( w_2 \).
b) What is the impact in the firm’s long-run supply function of a simultaneous and in the same proportion increase in \( p \), \( w_1 \) and \( w_2 \).
c) Find the conditional factor \( x_1 \) demand function.

Exercise 4
Mr. Bernardo is the coordinator of the production department in an international company. Each year, Mr. Bernardo receives information about the amount of production and the need to be economically efficient in a perfect competitive market.

The production function is described by \( Y = K^aL^b \).

The prices of capital and labor are \( r \) and \( w \), respectively.

a) Formalize the problem.

b) Find the conditional factor demand functions. What kind of information do they give?

c) Find the cost function and check its properties.

Felisberto is another coordinator in this firm. He received, though, a different task: to get the maximum profit possible. The production function and the market situation are similar to the previous ones.

d) Formalize the problem.

e) Find the non-conditional factor demand functions. What kind of information do they give?

f) Find the profit function using two different strategies. Check its properties.

g) Relate both problems. (Do not forget to interpret the sign of the derivatives).

5. Input Markets

Exercise 1

A certain firm produces a unique product using the production function \( Q = 2X^{0.5} \), where \( X \) is its unique input.

a) Suppose that in the output market, where the firm competes in prices with other firms, the equilibrium price is \( p = 3 \). The input market is also competitive and the equilibrium price is \( w = 1 \). Compute the optimal levels of inputs, production and profit.

b) Imagine now that the firm is monopolist in the output market where the demand is described by \( Q = 12 - 2p \). The inputs market is competitive and the equilibrium price is again \( w = 1 \). Compute the optimal levels of inputs, production, product price and profit.

c) Consider a new situation. The firm competes in prices in the output market where \( p = 3 \) is the equilibrium price. But, it is monopsonist in the input market, where the supply respects the condition \( X^*(w) = 9w^2 \). Compute the optimal levels of inputs, production, inputs price and profit.

Exercise 2

The production function of a certain firm which aims at maximizing its profit is given by \( y = 2L^{0.5}K^{0.5} \) where \( y, K \) and \( L \) are the number of units of output, capital and labor, respectively.
Suppose that, in the short-run, the input \(K\) is fixed at \(K = 1\), and its price is €5.

a) Consider that the prices of the output and the input \(L\) are fixed at €12 and €4, respectively. Compute the optimal levels of labor, production and profit.

b) Consider now that there is a monopoly in the output market, where the demand is given by \(p = 16 - y\). Compute the optimal levels of labor, production and price.

c) Consider again that the firm is price-taker with \(p = 12\). Nevertheless, now there is a monopsony in the labor market, where the supply is \(w = 2 + 0.5L\). Compute the optimal levels of labor, production and wage.

d) Now the firm is monopolist in the output market and monopsonist in the input market. The demand for the output remains the same \(p = 16 - y\) and the supply of labor is \(w = 2 + 0.5L\).
Compute the optimal levels of labor, production and prices.

Exercise 3

Imagine a firm B which is monopolist in the market of X and monopsonist in the market of Y, the unique input used to produce X. The production function is given by \(X = Y^{0.5}\). Beyond the costs with the acquisition of the inputs, the firm also bears a fixed cost of 10.

This firm sells its product in a market where the demand is described by the expression \(X = 10 - p\) and buys the input to firm A. Firm A is price-taker and produces Y using input Z which it buys at \(P_z = 0.5\).

Firm A uses 2 units of Z to produce 1 unit of Y and it has no other costs.

a) Given firm A’s cost structure and price-taker behavior it has an infinitely elastic supply function for Y at its marginal cost of production level. Using this supply function, find a relation between the total quantity of Y demanded by firm B and the price of Y.

b) Find the equilibrium values for \(p_x, p_y, X, Y, Z\).

c) How much will firm A offer in order to buy firm B? If the production function of firm A was not CRS but DRS, would it be able to offer more or less to buy B? (Hint: Consider only one period of production)

Exercise 4

Union A faces a demand curve of workers per hour such that \(w = \sqrt{L}\), while union B faces a demand \(w = L^2\).

Which of the unions can raise total income of its members (wage times workers per hour) more easily if there is a minimum wage higher than the equilibrium?
Exercise 5
The firm Y uses high-qualified labor as its unique input. It produces Y according to the function \( Y = L \), where Y is the output and L is the level of labor used. Only the professional association can guarantee the quality of the workers. Thus, only those with its license can be hired by firm Y.

Firm Y is one among several firms that employ these workers, hence it behaves as a perfect competitor. But it has a monopoly in the output market facing the demand function given by \( P = 25 - Y \), where p is the unit price of the output Y.

The professional association acts as a monopolist in the high-qualified labor market, and is able to discriminate wage rates among the employers. Currently, the contracts between the association and the firm Y state a wage rate of w.

The minimum wage for the workers to be willing to work is \( w = 1 \), but since the association is monopolist, it imposes higher wage rates.

a) For a certain level of the wage rate, what are the production and employment levels at firm Y? Compute the profit.

b) Compute the wage rate the professional association will demand for each high-qualified worker employed in firm Y. Compute the equilibrium levels for the levels of employment, production, price, profit of Y and surplus of the association.

c) Compare the employment levels found in the previous question with those in the solution without the association.

d) Firm Y proposed the association a different mechanism to remunerate workers, where the level of employment in the solution without the association was assumed. The firm Y would pay a fixed amount to the association apart from the wage rate to each worker. In which condition will the association accept the proposal?

Exercise 6
Firm \( M_X \) is the unique producer of an intermediate product X which it produces with a marginal cost of 2, equal to the average cost.

There are 100 firms, \( C_{i}^{p} \), \( i = 1, \ldots, 100 \), all similar, which buy a quantity \( X_i \) of product X at the market price \( P_X \). They use X as an input to produce the output \( Y_i \) according to the production function \( Y_i = X_i^{0.5} \). Beyond the cost of the inputs, these firms also have a transformation cost of 5 per unit of output.

These 100 firms sell its product \( Y_i \) in a perfectly competitive market facing the inverse demand function \( P_y = 135 - 2Y \), where Y is the total demanded quantity.

Assume that firms are acting in the short-run and, thus, firms \( C_{i}^{p} \) might have positive profits.
a) Find the demand function for the input \( X_i \) of each firm \( C_{p_i} \).

b) Compute the equilibrium price of the input \( X \).

c) Compute the equilibrium price for output \( Y \).

d) Find the optimal quantities for \( Y \) and \( X \).

e) Assume that the government decides to regulate firm \( M_X \)’s activity. It allows the firm to have a profit margin no higher than 50% of its marginal cost.

   Find the impact in the profit of firms \( C_{p_i} \). Comment.

f) Without making any calculations, describe the effect in the profits of the monopolist and in the consumers’ surplus of this public measure.

**Exercise 7**

Producer \( X \) is a monopolist in the market of output \( X \). It faces a demand curve \( X = 100 - P \).

To produce a unit of output \( X \), the producer uses 1 unit of input \( Y \) bought to producer \( Y \) at the price \( R \).

Producer \( X \) is the only buyer of product \( Y \). Besides the cost with the input, he has also a transformation cost of 8 per unit of output. \( Y \) is the unique producer of the input \( Y \) and the marginal cost of \( Y \) is constant and equal to 2.

a) Find \( p \), \( R \), \( X \), \( Y \) and the profits of each monopolist.

b) Assume that producer \( X \) acquired producer \( Y \)’s firm. Find \( p \), \( X \) and the profit of the resulting firm.

   Compare the results with those found in the previous question.

**Exercise 8**

In Microland, a small closed economy, there is only one firm that produces and sells cars, Monocar (M). It faces a demand curve for cars given by \( P_Y = 3500 - 0.5Y \), where \( Y \) is the total number of cars demanded.

In this economy there also 100 firms \( J_i \), with \( i = 1, 2, ..., 100 \), all similar, which produce wheels for cars.

Consider that each car has four and only four wheels when it is sold. The government’s protectionism forces Monocar to buy the wheels from this 100 firms which only sell to Monocar. Hence, Monocar is monopsonist in the wheels’ market and the producers are price-takers.

Each firm \( J_i \) has the following cost function: \( C_{T_i} = 25X_i \), where \( X_i \) is the number of wheels produced.

Each car produced by Monocar has a cost of 900 per unit, in addition to the cost of the wheels.

Assume that firms are deciding in the short-run, and thus the number of producers of wheels is constant.

a) Find the equilibrium price of the wheels and the cars, as well as the quantities.

b) The prices and quantities found before are equal to those in a situation where Monocar acts as a price-taker in the wheels’ market, i.e., if the price of the wheels was determined as in a competitive market. Why?
Exercise 9

In the Platenlands, the firm Maçarico (M) is the only producer and seller of catalizers for cars. It faces the demand curve \( P_Y = 450 - 2Y \), where \( Y \) is the total number of catalizers. Each catalizer uses platen in the proportion \( X = 4Y \), where \( X \) is the quantity of platen. Since the government has a highly protective international policy, firms act in a closed economy. There no other firms in the country that use platen, therefore Maçarico is monopsonist in the platen market.

There are 20 mines of platen, all similar, which act in the market as price-takers. Each mine has total costs of \( CT_i = \frac{X^3}{3} \) with \( i = 1, 2, \ldots, 20 \). Each catalizer produced by Maçarico has a cost of 50 per unit in addition to the cost of platen.

a) Find the equilibrium price for the catalizers and platen, as well as the quantities.

b) The government decided to open the market of platen. If the international demand is given by \( p_s = \frac{36}{X} \), how should Maçarico act in the new situation in order to maximize its profit? (Hint: Use graphical analysis and notice the similarities between this situation and one with minimum wage in the labor market)

c) The demand for catalizers abroad increased substantially due to environmental concerns and now it is \( p_s = 75 \). Find the total quantity of platen exported and the profits of Maçarico.

Exercise 10

A producer of good X has the following production function: \( X = 30L - 0.0125L^2 \), where L is the number of employed workers.

a) If the price of the output is €1, find the labor demand function.

b) Under the knowledge that this producer is monopsonist in the labor market and faces the supply curve:

\[
L_s(W) = \begin{cases} 
80W, & L \leq 800 \\
100W - 200, & L > 800 
\end{cases}
\]

Compute the level of labor used by the producer and the profit.

c) Consider a minimum wage of €12. Find the level of labor used by the producer in this new situation and its profit. Represent it graphically and interpret.

d) Consider now that the production technology changed to \( X = 19L \). Keeping the monopsony situation and the price of the output and ignoring the minimum wage, compute the new optimal production.

Exercise 11
Assume that the labor supply curve of a certain firm is $L'(w) = 100w$, where $w$ is the wage rate and $L$ the level of employment. The value of the marginal productivity of labor is given by $PMP(L) = 9 - 0.02L$.

a) If the firm is monopsonist in the labor market, how many workers will be hired and at which wage rate? And if there is a minimum wage of 2.5?
b) If the labor supply is monopolized by a union, how many workers will be hired in order to:
   i. Maximize the volume of labor?
   ii. Maximize the wage mass?
   iii. Maximize the workers surplus?

Exercise 12
There are two sectors in the economy, A and B. The labor demands are:

Sector A: $w = 50 - 5L_A$
Sector B: $w = 80 - 7L_B$

a) Let the equilibrium wage be $w = 5$. What is the total labor employed in each sector?
b) Assume that there is a minimum wage of 7.5 only in sector A.
   i. What is the new level of employment in sector A?
   ii. To absorb the excess labor supply, to which level must wage decrease in sector B?

Exercise 13
The manufacturing sector in a certain country consists of 100 similar firms. Their product is sold in a perfectly competitive market where there is perfect mobility of labor among the firms in the sector. All this mobility among other sectors and countries is already reflected in the labor supply functions.

The workers supply labor in a competitive market.

The production function for each firm is given by $X_i = 30L_i - 0.1L_i^2$, where $X_i$ is the output and $L_i$ is the input.

The labor supply functions for men and women are different, even though they are both equally productive:

$L_W = 10w_W + 200$
$L_M = 40w_M - 3200$

Furthermore, the price of the output is 10.

a) All markets are competitive. Find $w_W$, $w_M$, $L_W$, $L_M$ and X. Compute the excess total revenue over the total variable cost of the manufacturing sector.
b) Imagine now that the owners of the 100 firms constitute a union that acts as a monopsonist in the labor market. Compute the new values of the variables in the previous question.

c) Even though the manufacturing firms have monopsony power, the law forbids sexual discrimination. Compute the new levels for the variable in question a).

Exercise 14
There are 100 competitive firms acting in the market of output Q and using the labor (men and women) as an input. The price of output Q is €1 and the each firm’s technology is given by: \( Q_i = 25L_i \), where \( L_i = L_{Mi} + L_{Wi} \).

The labor supply functions for men and women are given by \( L_{Mi} = 10W_M \) and \( L_{Wi} = 40W_W \), respectively.

a) Consider a competitive market. Compute the equilibrium wage rate and the levels of labor for men and women. Find, for each firm, the level of labor used, production and profit.

b) Assume that firms constitute a union with monopsony power in the labor market. Find the new equilibrium and discuss the possibility of discrimination. Compute each firm’s profits in the new situation.

c) Due to a technological improvement, the production function changes to \( Q_i = 25W_M + L_{Mi} \).

Considering a competitive labor market, find the new equilibrium and explain the differences to the one found in a).

6. General Equilibrium

Exercise 1
Consider an initial endowment inside the Edgeworth box.

Can the negotiated contracts coincide to any point in the contract curve if the result is Pareto optimal? Comment.

Exercise 2

The contract curve has this name because it is the set of all possible contracts. Comment.

Exercise 3

To get a Pareto optimum it is necessary that the marginal rate of substitution between any two goods be equal for any two people that buy them. Comment.
Exercise 4
Represent graphically, and justify, the optimal Pareto allocations for two goods – tea (X) and coffee (Y) – consumed by two people, Mr. Lipton and Mr. Delta, in the following situations.

Assume that the endowment of coffee and tea are $\bar{\Delta}$ and $\bar{L}$, respectively.

a) Mr. Delta only likes coffee and Mr. Lipton only likes tea.
b) They both like coffee and tea. The two goods are perfect substitutes for Mr. Delta. Mr. Lipton likes consuming a certain quantity of tea that equals $\frac{\bar{L}}{\bar{\Delta}}$ of the quantity of coffee.
c) Mr. Delta and Mr. Lipton have preferences given by:
$$U_D = X_D + L_D \text{ and } U_L = 2X_L + Y_L$$
d) Mr. Delta and Mr. Lipton have strictly concave to the origin indifference curves.

Exercise 5
Assume that a certain consumer type A has $\bar{X}$ units of $X$ and a certain consumer B has $\bar{Y}$ units of $Y$. The utility functions of these two consumers are given by the expressions:
$$U_A = X_A Y_A \text{ and } U_B = X_B Y_B$$
where $X$ and $Y$ are the quantities consumed.

a) What is the equilibrium relative price of $X$ in a competitive market?
b) Is this equilibrium unique?
c) Define the contract curve.

Exercise 6
Two tribes, the Ocidentals and the Orientals, inhabit a certain island. They live in the respective sides of the island and meet only once a year in the annual market. No contact to the rest of the world is established. Both tribes collect corn (Y) and hunt deers (X). In the annual market, the deers are exchanged for corn in competitive conditions.

There are 1000 ocidental families and each produces 30 deers and 200 tons of corn. The utility function of each ocidental family is
$$U_{Oc} = X_{Oc}^{0.5} \cdot Y_{Oc}^{0.5}, \text{ where } X_{Oc} \text{ is the number of deers and } Y_{Oc} \text{ is the number of tons of corn consumed in a year.}$$

There are 2000 oriental families and each produces 25 deers and 300 tons of corn. The utility function of each oriental family is
$$U_{Or} = X_{Or}^{0.75} \cdot Y_{Or}^{0.25}, \text{ where } X_{Or} \text{ is the number of deers and } Y_{Or} \text{ is the number of tons of corn consumed in a year.}$$

The annual market achieves a competitive equilibrium.
How many deers are exchanged by 1 ton of corn? Each family consumes how many deers and how many tons of corn?

**Exercise 7**
Consider an economy without production where there are two agents, A and B, with the following endowments: \( X_A = 10 \), \( Y_A = 20 \), \( X_B = 20 \) and \( Y_B = 10 \).

Their utility functions are \( U_A = X_A^{0.6} \cdot Y_A^{0.4} \) and \( U_B = X_B^{0.5} \cdot Y_B^{0.5} \).

a) Using the data find the relative price interval in which both agents are willing to trade. Determine the exchange pattern: who sells \( X \) and who sells \( Y \).
b) Check if \( P_Y / P_X = 2 \) is a competitive equilibrium. Explain why and, if not, describe how prices will evolve?
c) How much is \( Y \) worth in terms of \( X \) in a competitive equilibrium? Represent the initial endowment and the competitive equilibrium in an appropriate diagram.
d) Assume that agent A’s preferences are now characterized by the following statement: “I must consume 2 units of \( X \) for each unit of \( Y \)”.  
i. Show that agent A’s marginal rate of substitution is not always defined.
ii. What is the shape of the contract curve?
iii. Given the property in i., this economy has a continuum of competitive equilibria. Comment.

**Exercise 8**
A certain economy produces only one consumption good \( X \) using only one input: labor \( Y \). Assume that there is only one type of families in this economy and consider each family’s decisions with respect to consumption and hours of labor in a certain period of 24 hours.

Each family’s preferences can be represented by the utility function \( U = XO \), where \( X \) measures the units of consumption good and \( O \) the hours of leisure. The profits in the production of \( X \) are totally distributed among the families in the same period.

a) The current technology is represented by the production function \( X = \sqrt{L} \) and there is competitive equilibrium in the economy. The equilibrium real wage rate is \( \sqrt{2}/8 \). Find the production level, the total number of hours of work used in the production of \( X \) and the profit. Check the families’ budget restriction.
b) There is an alternative technology described by the production function \( X = L \). Compute the equilibrium real wage rate and the profit if this technology is chosen. How much of good \( X \) will be consumed?

**Exercise 9**
Mr. Crusoe’s main problem when he was isolated in the island was to guarantee his subsistence. In the beginning he was able to survive eating fruits ($X$). The quantity of fruits he got depended on the time spent gathering them ($L_X$) according to the production function $X = L_X$.

Unfortunately, fruits only last one day. Hence, Mr. Crusoe must gather fruits every day. Each day, his utility function is given by $U = X(24 - L_X)$, where $L_X$ is the number of daily hours used gathering fruits.

a) How much time must Mr. Crusoe spend gathering fruits each day? How many fruits will he get this way?

After some time, Mr. Crusoe found out he was able to fish. If he uses $L_Y$ hours a day fishing, he gets the quantity $0.5 Y^{0.5}$ of fish. Considering the possibility of fish consumption, his utility function is now $U = (X + 4Y)(24 - L)$, where $L$ is the total quantity of time working.

b) Find the new optimal allocation for Mr. Crusoe? How many fruits and fish will he consume?

Exercise 10
In a certain economy there are two types of goods: agricultural goods ($X$) and industrial goods ($Y$). The production possibilities set in this economy can be described by $Y = 9 - X^2$ and the consumers’ preferences by the utility function $U = XY$. The government imposed the exchange of 1 unit of industrial goods by 4 units of agricultural goods.

a) What is the decision producers will make when facing this central restriction?

b) Is this a general equilibrium? Why?

c) If the government had let the market work by itself, would the price of the industrial good be higher or lower? Justify.

Exercise 11
In order to achieve Pareto efficiency it is necessary that the technical rate of substitution between any two inputs be equal for all firms that use positive quantities of them, even if firms produce different goods. Comment.

Exercise 12
The transformation curve of a certain economy is linear when all production functions have constant returns to scale. Comment.

Exercise 13
If the marginal rate of transformation between any two goods is not equal to the marginal rate of substitution between them for all consumers, then at least one of the goods is not being efficiently
Exercise 14
A certain country has two inputs, K and L, in fixed quantities. These inputs are homogeneous and there are no barriers to intersectorial mobility. Capital and labor are used in two activities, X and Y, with similar CRS production functions.

a) What is the shape of the contract curve in the space of the inputs? And the transformation curve? What is the inputs’ equilibrium price? Justify.

b) If there is only one consumer in this economy (consumer type A) whose preferences are homothetic and with \( MRS_A = \frac{4X}{Y} \), compute the ratio of the equilibrium quantities of X and Y.

c) Explain the change in the equilibrium prices of the inputs (with respect to those in b)), if, instead of consumer type A, the economy had only one consumer type B with \( MRS_B = \frac{2X}{Y} \).

Exercise 15
Two goods, X and Y, are produced in a certain country, with CRS production functions. There are 2 inputs in fixed quantities (\( \bar{K} = 100 \) and \( \bar{L} = 9 \)). All markets are competitive and with perfect intersectorial mobility of inputs. The inhabitants' preferences for goods X and Y can be represented by the marginal rate of substitution: \( \frac{4X}{Y} \).

a) In a certain period of time the equilibrium prices in this economy are:

\[
\frac{P_X}{P_Y} = 1, \quad \frac{W}{P_Y} = 50 \quad \text{and} \quad \frac{R}{P_Y} = 5.
\]

How much is produced of X and Y?

b) Having the information that the production function of Y is given by the expression:

\[ Y = 10\sqrt{10}K^{0.5}L^{0.5} \]
and that the productive efficiency implies \( k_\chi > k_\gamma \), represent graphically the transformation curve and compute the point of efficient production when only Y is produced.

c) Imagine that the production function of X changes to one represented by the same algebraic expression of Y and that the quantities of inputs remain the same. Compute the general equilibrium quantities for X and Y.

Exercise 16
Consider an economy with only two firms, X and Y, which use two inputs, capital (K) and labor (L), in the following way:

- The production function of X is described by \( X = L^x K_X \),
The production of Y uses fixed coefficients of inputs, i.e., always uses 10 units of labor for each unit of capital.

Assume that the producers of X have 30 units of labor and 6 units of capital, and those of Y have 20 and 4, respectively.

a) Represent, in an Edgeworth box in the space of the inputs, the contract curve for this economy. How does the relative wage \( \frac{w}{r} \) change along the curve when sector X expands?

b) Show that, starting with the initial endowments and in order to achieve a competitive equilibrium, sector X will expand its units of capital and transfer labor to sector Y and the relative wage \( \frac{w}{r} \) will equal 0.3, i.e., ten workers will cost three units of capital.

c) Assume now that labor is so specific to each production that it cannot be transferred between them. Starting at the initial endowment, and given the immobility of labor, what are the Pareto optimal allocations? Identify those allocations that would be efficient if labor was flexible. (Make a graphical analysis and justify it carefully)

Exercise 17

The Bedrock city, where Filinstons live, has no international economic relations. Two families live there: Fred’s family (Fred, Wilma and the baby) and Barney’s family (Barney, Betty and their daughter).

Everything in this city is rocky. There are two kinds of rocks: the hard ones (H), used to build houses, and the soft ones (S), used as food.

The utility of each family depends only on the consumption of these two types of rocks. While Fred’s family wants to consume 4 Kilo of Soft Rocks (S) for each Kilo of Hard Rocks (H), Barney’s family is more flexible in those proportions, having a utility function given by \( U_B = H_B^{0.5} S_B^{0.5} \).

Both families can extract both types of rocks without any cost, but in different quantities. Their initial endowments are:

\[
\begin{align*}
H_F &= 20 \text{ Kg} & S_F &= 50 \text{ Kg} \\
H_B &= 40 \text{ Kg} & S_B &= 100 \text{ Kg}
\end{align*}
\]

a) Check if the initial endowment is a Pareto optimum for the Bedrock economy. Justify.

b) Even though there are only two families, they agreed to exchange rocks in a competitive market. Find the equilibrium for this economy.

c) Starting at the equilibrium in b), the government of the country which Bedrock belongs to decided that each family must consume at least 75 Kilo of Soft Rocks (S).

i. The government advocates that this measure implies a Pareto improvement. Comment.

ii. For which conditions does this kind of measure imply an increase in social welfare?

iii. If the measure implies an increase in social welfare, is it possible to find other measures that imply a bigger increase in social welfare?
d) Assume that Barney’s family can no longer extract Soft Rocks (S) costlessly. They must use 0.5 Kilo of Hard Rocks (H) to extract 1 Kilo of Soft Rocks (S).

i. Explain why the equilibrium price for Hard Rocks must be 2.

ii. Verify that the equilibrium consumption allocations for both families are:

<table>
<thead>
<tr>
<th></th>
<th>Family F</th>
<th>Family B</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>15 Kg</td>
<td>20 Kg</td>
</tr>
<tr>
<td>S</td>
<td>60 Kg</td>
<td>40 Kg</td>
</tr>
</tbody>
</table>

**Exercise 18**

In a certain isolated island there are two families (1 and 2), which exchange, in a competitive way, food (A), collected from nature, and drinkable water (W) that each family gets from the closest river.

a) If both families have utility functions given by \( U_i = A_i^a W_i^{1-a} \), with \( l = 1, 2 \) and \( 0 < a < 1 \), what is the shape of the contract curve? Represent graphically and interpret.

b) For \( a_1 = 1/3 \), \( a_2 = 2/3 \) and initial endowments \( (A, W) = (10, 5) \) for family 1 and \( (A, W) = (10, 5) \) for family 2, find the contract curve and represent it graphically.

c) Find the offer curve for family 1 and compute the general equilibrium.

To get these two goods, each family must spend their available daily time. The more food a family wants to get, the bigger the distance it must make, thus, the smaller the quantity collected in each hour. Since the river is always at the same distance, each litre of drinkable water demands the same number of hours.

Unfortunately, last week, a strong storm affected the region where family 1 lives and now they must climb the mountains to get food, spending much more time.

d) Determine the impact of the storm in the equilibrium price. Represent the changes in the Production Possibilities Frontier of the island and the equilibrium points before and after the storm.

**Exercise 19**

In a certain economy the initial endowments of inputs, capital and labor, are \( \overline{K} = 100 \) and \( \overline{L} = 100 \). These inputs might be used in the production of two goods. Good X is produced respecting the function \( X = L_X^{0.5} K_X^{0.5} \), and good Y is produced respecting the function \( Y = \ln K_Y + 2 L_Y^{0.5} K_Y^{0.5} + \ln L_Y \).

The two goods are perfect substitutes to the consumers, to whom 5 units of Y give as much utility as 2 units of X.

a) Find the contract curve in the Edgeworth box for inputs. Comment.

b) Find the general equilibrium solution. (Hint: First find the Production Possibilities Frontier, noticing that the production function for Y does not have constant returns to scale).
7. Welfare Economics

Exercise 1
There is a Pareto improvement if a certain economy changes from a point inside the Utility Possibilities Frontier (UPF) to a point in the frontier.

Exercise 2
We want to divide a cake between two people. Pareto optimality means that each receives half of the cake.

Exercise 3
Xavier and Zacarias have similar utility functions: \( U(x, y) = x^2 + y^2 \). There are 10 units of x and 10 units of y to be divided among them.

a) Represent their preferences and the contract curve in an Edgeworth box.

b) Define the fair allocations.

Exercise 4
Comment the following statements:

a) According to Arrow’s Impossibility Theorem, it is impossible to sort social preferences such that they are complete, reflexive and transitive.

b) A certain allocation is fair if, when one person envies another, the latter does not envy the former.

c) In a pure exchange economy, if an allocation is Pareto efficient, it is impossible to have two individuals that prefer the others’ consumption bundle to his own.

Exercise 5

a) If the Social Welfare function (SWF) is equal to the sum of individual utilities, when one divides a fixed amount of money, the SWF is maximized if there is an equal division.

b) In which circumstances does an equal division correspond to the maximum social welfare?

Exercise 6

One can express social preferences voting by ordered results.

a) Verify that this social preferences relation is complete, reflexive and transitive. Justify.

b) If all consumers prefer x to y, is x socially preferred to y?

c) Does the social preferences relation respect the independence of irrelevant alternatives condition?

Exercise 7
When the paintings were found at Foz Coa, there was a great discussion about building the planned dam. Different entities have different preferences with respect to the decision to make.
Assume that the three entities involved – archaeologists (A), EDP (B) and inhabitants (C) – select a representative to attend a decision meeting. Four proposals are at the table: to continue the construction not caring about the paintings (W), to stop the construction (X), to continue the construction protecting the paintings which might be visited (Y) and to continue the construction and collecting the rocks which would be part of a big tourist park (Z).

The three parties have different preferences with respect to the alternatives which they order in the following way (decreasing order of preference):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
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<tbody>
<tr>
<td>X</td>
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<td>W</td>
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a) Assume that the representatives must vote in one of the alternatives in a majority voting system. If each individual votes in the most-preferred option, what would be the final decision?

Due to the problem detected in a), they decided to change the voting system to one of ordered results. Each representative gives 1 point to the most-preferred alternative, 2 to the next and so on. The chosen option would be that with fewer points.

b) If each representative attributes the points directly according to his preferences, which one will be chosen?

c) EDP would be specially disappointed with the result found in b). Explain how, acting strategically, its representative would change the result to a better one from EDP’s point of view.

**Exercise 8**

a) Verify if the social choice using the voting system always implies Pareto potential improvements.

b) Consider a society where each inhabitant has 1 vote. The voting right can be competitively exchanged between them. Explain if this system of social choice always accepts alternatives that are Pareto potential improvements.