EXERCISES
PART I

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1. Introduction

1.1 Preferences and utility function

Exercise 1
Suppose you ask someone to choose between a computer and two boxes of JB whisky. Which of the following answers violates the axioms of consumer preferences?

a) “I don’t know”
b) “They are so different that I cannot choose”
c) “One or the other”
d) “I don’t care, you choose”

Exercise 2
Consider all the students attending the classes on Microeconomics II and the relation “is at least as good as”. Is this relation complete, reflexive and transitive? Repeat the exercise using the relation “is better than”.

Exercise 3
Filipe Escolar is the coach of a popular soccer team and he must choose the striker that will play the next game. The player he will choose must combine three qualities: good physical condition, ability to score and tactical discipline. The following table classifies the three candidates for the place, according to those qualities:

<table>
<thead>
<tr>
<th></th>
<th>Liedson</th>
<th>Sabrosa</th>
<th>McCarthy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical condition</td>
<td>Excellent</td>
<td>Reasonable</td>
<td>Very bad</td>
</tr>
<tr>
<td>Ability to score</td>
<td>None</td>
<td>Excellent</td>
<td>Reasonable</td>
</tr>
<tr>
<td>Tactical discipline</td>
<td>Medium</td>
<td>None</td>
<td>Excellent</td>
</tr>
</tbody>
</table>

Filipe Escolar prefers player A to B if player A is better than player B in at least two of these three qualities. Is it true that Filipe Escolar has transitive preferences? Discuss.

Exercise 4
Consider the following utility functions:

(1) $U = x_1 x_2$
(2) $U = x_1^2 x_2^2$
(3) $U = x_1^2 x_2^3$
(4) $U = 4x_1 x_2$
(5) $U = 4x_1 x_2 + 17$
(6) $U = \log(x_1 x_2)$

Identify those that are monotonic transformations of (1).
1.2 Consumer’s choice and demand

Exercise 1
A certain consumer spends €400 weekly in the acquisition of goods x, y and z. The market prices are €20, €80 and €50, respectively.
   a) Find the consumer’s budget line.
   b) Represent it graphically.

Exercise 2
Suppose that \( U \) is an utility index which allows a certain consumer to order, in terms of preferences, different combinations of goods \( x_1 \) and \( x_2 \):
\[
U = 2x_1^{0.5}x_2^{0.5}
\]
The consumer has $600 and the prices of goods 1 and 2 are €30 and €15 respectively.
   a) Define the budget line analytically.
   b) Find the bundle the consumer will choose. What is the marginal rate of substitution at the optimum? Represent graphically.

Exercise 3
   a) Find the optimum for a consumer whose utility function and budget line are
      \[
      U = x_1^{1.5}x_2 \quad \text{and} \quad 3x_1 + 4x_2 = 100
      \]
   b) Suppose that now the utility function is given by
      \[
      U = x_1^6x_2^4 + 1.5 \ln x_1 + \ln x_2
      \]
      keeping the same budget line. Show that the optimal bundle is equal to the one found above. Justify.

Exercise 4
The utility index \( U_A = x_2^{15} \) allows consumer \( A \) to order, in terms of preferences, different combinations of goods \( x_1 \) and \( x_2 \). Using the knowledge that \( P_1 = 2 \), \( P_2 = 4 \) and \( m = 10 \), find the optimum for consumer \( A \) and represent it graphically.

Exercise 5
Suppose that \( U \) is a utility index which allows a certain consumer to order, in terms of preferences, different combinations of goods \( x \) and \( y \):
\[
U = 4xy + 4x
\]
The indifference curves associated to this utility index \( U \) are strictly convex to the origin. The consumer’s income is €1300 and the market prices of goods \( x \) and \( y \) are €30 and €50, respectively.
   a) Find the optimal bundle for this consumer. Represent the problem’s solution graphically.
   b) Suppose that, due to a big economic crisis, the government decides to impose restrictions on the consumption of goods \( x \) and \( y \): each consumer must not consume more than 20 units of each good; 30 coupons are given to each consumer and, in
addition to the monetary payment, for each unit of good x or good y bought one coupon must be handed over. Find the chosen bundle in the new conditions.

c) Repeat the last question using an income of €1150.

**Exercise 6**

The income of a certain consumer is €120 and it is totally spent in the consumption of goods x and y. The price of y is €3. The price of x is not constant since it depends on the quantity bought.

Hence,

- If $0 \leq x < 20$, the price of x is €4.
- If $20 \leq x < 40$, the price of x is €3.
- If $x \geq 40$, the price of x is €2.

Find the optimum for each of the following preferences relations:

a) $U = \min \left[ \frac{x}{3}, \frac{y}{4} \right]$

b) $U = \min [x, y]$

c) $U = \min \left[ \frac{x}{4}, y \right]$

**Exercise 7**

Consider the demand curve $q = p^{-1.5}$.

a) Calculate the effect over total expenditures of a decrease in price from €4 to €3.

b) Recalculate for $q = p^{-0.4}$.

c) Explain the changes in the total expenses using the concept of price elasticity of demand.

**Exercise 8**

Consider the demand curve:

$q = a - bp$, with $a, b > 0$

Find the expression that allows you to calculate the price elasticity of demand in each point of the line.

**Exercise 9**

Consider a consumer whose preferences are given by the utility index $U = 2x_1^{0.7}x_2^{0.3}$, where $x_1$ and $x_2$ are the quantities he consumes of goods 1 and 2.

a) Find the analytical expression of the ordinary demand function of each good.

Consider that the prices of goods 1 and 2 are €20 and €25, respectively.

b) Find the analytical expression of the income offer curve e represent it graphically. Calculate the optima for this consumer using the income levels €300 and €500.
c) Find the Engel curve for each good. Represent the curves graphically.

Consider the income level €300.

d) Find the demand curve for each good and represent them graphically.

e) Find the price offer curve of good 1. Use the income level of €300 and the price €25 for good 2. Interpret.

f) Consider the following situations:

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<table>
<thead>
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</thead>
<tbody>
<tr>
<td>P₁</td>
<td>P₂</td>
<td>m</td>
</tr>
<tr>
<td>A</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
<td>25</td>
</tr>
</tbody>
</table>

- Use the ordinary demand curves to calculate the optimal bundles for the consumer in each situation.
- Use the relevant demand curve to find the quantities of good 1 bought in situations A and C.
- Use the relevant Engel curve to find the quantities of good 1 bought in situations A and B.

Exercise 10

Consider the following utility function:

\[ U(x, y) = (x + 1)^{0.4} \cdot (y + 2)^{0.6} \]

a) Find the demand functions of goods x and y.

b) Calculate the income elasticities of demand for goods x and y.

c) Calculate the direct and cross price elasticities of demand.

d) Represent the income expansion path for \( p_x = p_y \).

e) Represent the Engel curve of good x for \( p_x = p_y = 1 \).

f) Find the price offer curve of good x for the price \( p_y = 1 \) and income €10.

Exercise 11

Consider a certain consumer that spends all his income in the consumption of goods 1 and 2. The following graph represents his price offer curve of good 1.
a) Classify good 1 in terms of direct price elasticity of demand, 1_1, in each of the five points above.

b) Identify the sign of the cross price elasticity of demand, 2_1, in each of those points.

**Exercise 12**

Suppose that in Beja the consumers pay for a kilo of pork the double of what they pay for a kilo of turkey; and in Odemira the consumers only pay for the pork half the price of the turkey. Assuming that they are all maximizing their utility, what is the relation between the marginal rates of substitution of these two goods in both cities?

**Exercise 13**

Consider the following table with information about the marginal utility of the bottles of whisky and gin to Mr. Smith and Mr. Bacon. At the moment, Mr. Smith has two bottles of each beverage and Mr. Bacon has one bottle of whisky and two of gin.

<table>
<thead>
<tr>
<th>Mr. Smith</th>
<th>Mr. Bacon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whisky</td>
<td>Gin</td>
</tr>
<tr>
<td>Nr.</td>
<td>Marginal utility</td>
</tr>
<tr>
<td>1</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>Nr.</td>
<td>Marginal utility</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Is it true that Mr. Smith want to exchange a bottle of whisky for a bottle of gin? And one bottle of whisky for two of gin? Would Mr. Bacon accept the exchanges?

**Exercise 14**

A certain couple has €10 to spend in houseware (good x) and other things (good y). The price of both good x and good y is the same and equal to 1. The only information we have about this couple is the one in the following graphs:

![Graph](image)

a) Find the bundle that maximizes the couples' utility in this situation and in that when m=3.

b) Consider again that m=10. Classify both goods using the information about their income elasticities of demand. Calculate those elasticities and find their values if m<4.
Exercise 15

a) Represent the price offer curve of butter for the following consumers who have an income $m$ to spend in butter ($y$) and jam ($x$). Their preferences are stated in the following sentences:

i) “For me, it is totally indifferent to eat bread only with butter, only with jam or with any proportion of both.”

ii) “I can only eat bread with both butter and jam, since this way the bread is neither too sweet neither too fat”

Explain.

b) Consider now that for another consumer with an income of €10 and $p_x=1$, one could get the price offer curve (POC) of good $y$: $x=\frac{26y+30}{5y+3}$

i) Using the POC given, find the price of $y$ so that this consumer wants to buy two units of good $y$.

ii) Using the POC given, find the expression of the ordinary demand curve for $y$. Represent it graphically and relate it with the POC.
1.3 Substitution and Income Effects

Exercise 1
Consider the utility function $U(x, y) = x^{0.5} y^{0.5}$ and the following situations:

A: $p_x = 1; p_y = 2; m = 41$

B: $p_x = 1; p_y = 4; m = 41$

a) Find the ordinary demand functions of goods x and y.
b) Decompose the total change in demand (good y) in the Slutsky’s substitution effect and in the income effect. Do the graphical and the analytical analysis.

Exercise 2
John and Tony are good friends and they spend all their monthly income in beers (x) and cinema (y). They have the same preferences for these two goods,

$$U = -\frac{1}{x} - \frac{1}{y}$$

a) They both receive a monthly income of €60 and the price of a cinema ticket is €5, the same as a beer. Find how many times they go to the cinema and how many beers they drink a month.
b) The price of a cinema ticket increased and now it costs €20. What is the effect of this change in the number of times they go to the cinema? Decompose the total change in demand using the Slutsky’s criterion. Use a graph to support your answer.
c) Due to the big increase in the price of the tickets, John and Tony tried to convince their parents to increase the monthly income. Their parents did increase it so that they could drink the same number of beers and go to the cinema the same number of times as before. Nevertheless, John’s father, stricter than Tony’s, forbade him from drinking more beers than initially. John protested once more, saying that he would lose Tony’s company in some cinema sessions. Is he right? Justify your answer with all the necessary calculations and using a graphical representation.
d) Calculate John and Tony’s utility in the previous question and interpret the values found.

Exercise 3
Which of the following statements are true? Justify.
a) “A certain good is inferior if and only if the demand for this good decreases when price decreases”
b) “To help the families with lower income, the government decided to subsidize the first 100KW of energy consumed and to tax ad valorem the remaining units. If for a certain family the level of utility remains the same after this decision, this means that the amount of taxes paid cannot exceed that of subsidies received.” (Hint: make a graph.)
c) “Whenever the marginal rate of substitution is decreasing, the marginal utilities are negative”
d) Consider the following utility function: \( U(x_1, x_2) = V(x_1) + x_2 \), where \( V'(x_1) < 0 \) and \( V''(x_1) < 0 \). If the price of good 1 decreases, there will always be a positive income effect in the demand of good 1.

e) “Suppose that for a certain consumer of wine and cheese the marginal utility of the last bottle of wine is 5 and the marginal utility of the last kilo of cheese is 10. If a bottle of wine costs €90 and a kilo of cheese costs €10, he must increase the consumption of cheese and reduce the consumption of wine in order to maximize his utility.”

Exercise 4
Consider a consumer of goods 1 and 2 whose income is €300. We know that, for the market prices \( P_1 = €20 \) and \( P_2 = €25 \), he chose the bundle \( A = (x_1; x_2) \) with \( A = (7.5; 6) \).

Suppose that the price of good 1 changes to €15. Represent graphically both budget lines. If the consumer is taxed such that with the new vector of prices he would be in a situation of indifference, he would choose the bundle \( B = (11; 4) \).

Discuss the substitution and income effects for the following final optimal bundles:

a) \( C = (7.5; 7.5) \)

b) \( D = (12.5; 4.5) \)

c) \( E = (3.33; 10) \)

Exercise 5
“When the price changes, the total change in demand is always higher than the substitution effect, i.e., the ordinary demand curve is always flatter than the compensated demand curve”. Discuss.

Exercise 6
Consider the utility function \( U = x^{0.5} y^{0.5} \).

a) Find the ordinary demand curves of goods x and y.

b) Consider the following situations:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_x )</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( P_y )</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( M )</td>
<td>41</td>
<td>41</td>
</tr>
</tbody>
</table>

Decompose the total change in the demand of good y caused by the change in its price into a Slutsky’s substitution effect and an income effect. (Hint: Use the functions found in a))

c) Find the compensated demand functions of goods x and y.

d) Repeat b), using Hicks’ compensation criterion. (Hint: Use the functions found in c))

Exercise 7
Repeat the previous exercise using the utility function \( U = 0.3 \ln x + 0.6 \ln y \) and the following situations:
Exercise 8

Is the following statement true or false? Justify.
“A certain consumer buys good x in a certain store and good y in another store. To purchase good y he must use the bus. If the bus fare increases, the consumer buys less of both goods given that none of them is inferior and both are still bought.”

Exercise 9

A certain consumer spends all his weekly income in goods x and y. His preferences are expressed by the function $U = x^y$ and, consequently,

- the demand function of good x is given by $x = \frac{m}{2Px}$, where m is the weekly income and Px the price of a unit of x.

- the compensated demand curve of good s is given by $x = \left(\frac{Up_y}{Px}\right)^{1/2}$, where Py is the price of a unit of good y.

Consider also that m = €24, Px = €4 and Py = €1.

a) Find the optimal weekly quantities of both goods.

b) Suppose that the price of good x increased to €9 and that the consumer’s income was adjusted to $m' = 9x^* + y^*$, where $x^*$ and $y^*$ are the optimal quantities found in a). What are the new optimal quantities?

c) In alternative, consider that, in order to face the increase in the price of x from €4 to €9, the consumer’s income is adjusted such that the utility index in a) is not changed. Find the optimal quantities of x and y under this conditions.

Explain, briefly, which of the situations a), b) and c) will the consumer prefer. Use the compensation criteria implicit in b) and c).

Exercise 10

Imagine that a certain consumer of two goods wants to know the percentage change in x1 caused by an increase of 1% in p2.

He knows that 40% of its income goes to the purchase of good 2 and that when his income increases, the consumption of 1 and 2 increases by 60% and 159, (9)%, respectively. Moreover, when p1 increases by 1% the consumption of good 2 decreases by 0.4%.

Is it possible to classify the goods in terms of cross price elasticity of demand?

Do you think that the classification would change if, instead of the ordinary demand functions, the compensated demand functions were those used in the calculus of the elasticity?
2. Duality Consumer Theory

Exercise 1
The following utility function (quasi-linear) is characterized by not having an income effect for one of the goods:

\[ U = \alpha \ln x_1 + x_2, \ para \ y \geq \alpha p^2, \]

where \( y \) is the income.

a) For which of the goods there is no income effect? Represent the income expansion path.

b) Solve the consumer’s primal problem in order to find the marshallian demand functions \( x_i (p, y) \) and the indirect utility function \( v (p, y) \).

c) Find the price offer curve of good 1 and represent it. (Do not forget the restriction \( y \geq \alpha p^2 \).)

d) Find the hicksian demand functions without solving the dual problem.

e) Construct a table with the solutions of the primal and the dual.

Exercise 2
Consider the following utility function:

\[ U = x_1^\gamma x_2^{1-\gamma} \]

a) Derive the indirect utility function and the expenditure function, solving the consumer’s primal and dual problems.

b) Verify Shephard’s lemma and Roy’s identity.

c) Calculate the income elasticity of demand and the price elasticity of demand. Are these results realistic?

d) Find to which do the shares \( w_i = \frac{p_i x_i}{y} \) correspond.

e) Find the income offer curve and the price offer curve of good 1. Represent graphically.

f) Verify the homogeneity of degree zero of the demand functions and the aggregation conditions of Engel and Cournot.

Exercise 3
Suppose that there are only two goods. The marshallian demand functions are given by:

\[ x_1 (p, y) = \frac{y}{p_1 + 2p_2} \]
\[ x_2 (p, y) = \frac{2y}{p_1 + 2p_2} \]

The prices are \( p_1 = €1 \) and \( p_2 = €2 \) and the income \( y = €50 \).

a) Show that the marshallian demand functions are homogeneous of degree zero in prices and income.
b) The price of good 1 increased to €2 but the consumer received a lump-sum subsidy which allow him to access the initial level of utility. In this new situation, what is the consumer’s demand for good 1? Explain the result.

**Exercise 4**

Consider the information about the demand functions and the patterns of expenditures of a consumer who spends all his income in two goods:

1. At the current prices, the consumer spends five times more in good 2 than in good 1;
2. At the current prices, the price elasticities of demand for goods 1 and 2 are equal to –3 and –2.

   a) Find the cross price elasticities of demand for both goods.
   b) What is the percentage change in the demand of both goods if the income increases 1%?

**Exercise 5**

Suppose that there are only two goods. Initially, a consumer with an income \( y = 10 \), facing the vector of prices \( (p_1, p_2) = (2,4) \) decided to purchase 1 unit of good 1 and 4 units of good 2. Afterwards, with an income of \( y = 15 \), facing the vector of prices \( (p_1, p_2) = (3,6) \), the same consumer chose the bundle \( (x_1, x_2) = (3,1) \). The consumer’s preferences did not change in this period of analysis.

Is the consumer maximizing his utility?

**Exercise 6**

A certain consumer has the following expenditure function:

\[
E = p_1^a \cdot p_2^b \cdot U
\]

a) What is the sign of \( a \) and \( b \)?

b) Knowing that \( U = \sqrt{LC} \), find the value of \( b \).

**Exercise 7**

a) Derive the aggregation condition of Engel when there are \( n \) goods.

b) Discuss the truthfulness of the following statements:
   i. “If one of the goods is inferior the other must be a luxury good. Conversely, if one of the goods is a luxury good the other must be inferior.”
   ii. “If all the income elasticities of demand are constant and equal, they must be equal to one.”

**Exercise 8**

Mr. Rigidity has Leontief preferences described by the following utility function:

\[
U = \min(ax_1, bx_2), \ a, b > 0
\]

a) Find the ordinary and the compensated demand curves, the indirect utility function and the expenditure function.
b) Find the income and the price offer curves of good 1. Represent them graphically.

Exercise 9
The preferences of Mr. Substitute are described by the following utility function:

\[ U = ax_1 + bx_2, \quad a, b > 0 \]

a) Find the ordinary and the compensated demand curves, the indirect utility function and the expenditure function. (Hint: start solving the problem graphically)

b) Represent graphically the income and price offer curves of good 1.

Exercise 10
Consider the following expenditure function:

\[ e = p_2 (v - \ln p_2 + \ln p_1 + 1) \]

With \( p_1 = €5 \), \( p_2 = €5 \) and \( m = €10 \), find the optimal quantities of both goods.

Exercise 11
Suppose two consumers have the same income and choose the same utility maximizing consumption bundle. They have different substitution elasticities, however. Now, suppose that the price of X increases and that they also both choose the same utility maximizing bundle.

Show graphically that the consumer with the smaller substitution effect requires the larger income subsidy to restore him to his original indifference curve after the price increase.

Exercise 12
Why is it that the price elasticities of the ordinary demand are not too different from the price from the price elasticities of the compensated demand?
3. Topics in Consumer Theory

3.1 The supply of labor

Exercise 1
John must choose the number of daily working hours he will offer his potential employer operating in Lisbon. The hourly wage rate is \( W_1 \) and his extra-wage income is \( X_1 \). John’s preferences are described by the utility index \( u(C, L) \), where \( C \) is the quantity of a composite good and \( L \) is the daily number of hours of leisure.

John’s indifference curves are strictly convex to the origin and the price of the composite good is 1. John has 16 hours to spend in work or in leisure.

a) Suppose that John decides to work 7 hours. Represent the budget line and the optimal choice.

b) Linda, John’s girlfriend, adviced him for the fact that the daily expenditures with transports to go to work are \( Z_1 \), which reduce his available income. Certainly these expenditures would be zero if he decides not to work. Graphically analyze the effect these expenditures have in the optimal number working hours. Can John decide not to work?

c) John received a proposal to work in another firm, which operates in Setúbal. This firm pays an hourly wage rate of \( W_2 > W_1 \) and the expenditures in transports are \( Z_2 \), with \( Z_2 > Z_1 \). If John is indifferent between the two jobs, in which of them will he work more? Fundament your answer using graphical analysis.

d) Ignore the expenditure costs and the job in Setúbal. Consider now that there is a minimum of 8 hours to work. Does John prefer to work 8 hours rather than not to work? Represent the new situation and compare the utility level with that in a).

Exercise 2
Discuss the following statement: “If leisure is a normal good, then an increase in the wage rate necessarily implies an increase in the number of hours an individual is willing to work”.

Exercise 3
The preferences of Mr. Happiness between leisure (\( L \)) and consumption goods (\( C \)) are described by the following utility index:

\[
U = \ln L + \ln C
\]

Mr. Happiness has a time endowment \( T \) to use in leisure and work. His hourly wage rate is \( W \) and his income from financial assets is \( X \). Assume that \( WT > X \).

a) Find the labor supply curve of Mr. Happiness.

b) Represent graphically the relation between the number of working hours and the wage rate.

c) Find the wage rate that would induce Mr. Happiness not to work.

d) Calculate the income elasticity of demand.

e) Find the income \( X \) that would induce Mr. Happiness not to work given that \( W=W^* \).

Exercise 4
Mr. Smith’s daily preferences between hours of leisure (\( L \)) and consumption of goods (\( C \)) are described by the following utility index
\[ U = \sqrt{LC} \]

There is no extra-wage income, the wage rate is €10 and the time endowment to use in leisure and work is 16 hours.

a) Find the daily number of hours Mr. Smith decides to work.

b) Some changes occurred in Mr. Smith's earnings. Now, after 6 hours of work the wage rate goes to €15 (extra-hours). Find the new number of hours of work Mr. Smith decides to offer in the new circumstances.

**Exercise 5**

Miss Penny is an accountant earning €20 for each hour of work and able to work a maximum of 60 hours a week (time endowment). Recently, she received an inheritance from her uncle in the US. which she applied in investment funds. Each week, she gets €200 of interests which she entirely use in the consumption of goods.

Miss Penny’s preferences between consumption (C) and hours of leisure (L) are described by the following utility index:

\[ U = C^{0.6} L^{0.4} \]

a) Find the number of hours Miss Penny decides to work and the amount she will spend in consumption goods.

b) Suddenly, Miss Penny realized that she was not paying the due taxes. To avoid problems, she went to the Fiscal Department and, there, she concluded that all her earnings (i.e., the wages and the interests) are subjected to the following taxes: exemption until a weekly earning of €200, a marginal rate of 25% to the weekly earnings between €200 and €750 and a marginal rate of 50% to the earnings higher than €750. As an example, for a weekly earning of €1000, the amount of taxes would be (0*€200) + 0.25. (€750 - €200) + 0.5. (€1000 - €750) = €262.5.

Find the optimal number of working hours in these new circumstances.

**Exercise 6**

Melissa heard of a job as lifeguard in the beaches of Guincho, with total freedom to choose the number of hours of work. The hourly wage is €10 for the first 8 hours and €15 for each extra hour after the 8 hours and to a maximum of 14 hours a day.

Under these conditions, Melissa decided to offer 10 hours a day.

Before making the decided offer to the employer, she was informed that the wage had changed to a uniform rate of €11. (no distinction between normal and extra-hours).

Will Melissa change her decision? Explain.

Which of the two schemes is more attractive to Melissa?

**Exercise 7**

In a certain country, the wages are exempted from taxes before a certain amount and taxed at a fixed rate after it. Assuming that leisure is a normal good, can one say that an increase in the exempted amount cannot cause an increase in the number of hours of work?

**Exercise 8**

A certain individual behaves in a perfectly competitive way both in the labor and in the goods markets. He needs at least 8 hours to sleep, eat, etc. The remaining 16 hours can be spent working or resting. His utility function for those 16 hours is given by:
\[ U = (C \cdot L)^{\frac{1}{2}} \]
where \( C \) is the quantity of goods consumed and \( L \) is the number of hours resting. Define \( H \) as the number of hours he decides to work, \( P \) as the price of \( C \) and \( W \) as the hourly wage rate.

a) Define the consumer’s labor supply. Calculate the labor supply elasticity when \( P = 1 \) and \( W = 10 \).

b) The consumer must pay a tax over hourly earnings of \( T = 0.25 \cdot (Y - 60) \), where \( Y \) is the value of the earnings before taxes. How much will he choose to work with \( P = 1 \) and \( W = 10 \)?

c) If there is no tax, the labor contract changes. He receives 150% of the wage rate for each additional hour worked after the 6 hours a day. How much will he choose to work with \( P = 1 \) and \( W = 10 \)?

d) Suppose that he has the right to have access to extra-hours defined as in c), but in this case he must pay the tax in b). Again, how much will he choose to work with \( P = 1 \) and \( W = 10 \)? Is the consumer better with extra-hours and taxes?

**Exercise 9**

Mr. Only Rest has Cobb-Douglas preferences between leisure (\( L \)) and consumption (\( C \)):

\[ U = L^\alpha C^{1-\alpha} \]

He receives a monthly income of \( X \) from his rich uncle and \( W \) for each hour of work. He has \( T \) hours to use in work and leisure.

a) Find the labor supply of Mr. Only Rest and derive the necessary condition for \( H > 0 \). (\( H \) is the number of hours working)

b) Represent graphically the labor supply curve of Mr. Only Rest, defining its reservation wage.

c) Calculate the elasticities of labor in terms of wage and monthly exogenous earnings.
3.2 Intertemporal Consumption

Exercise 1
The family Ferreira has intertemporal preferences described by the following utility index:

\[ U = C_1^{0.6} C_2^{0.4} \]

where \( C_1 \) and \( C_2 \) are the expenditures in consumption goods in periods 1 and 2, respectively. This family has an income of 100 in each period (i.e. \( M_1 = M_2 = 100 \)). The strong friendship with Mr. Lima allows Mr. Ferreira to borrow from him at a constant interest rate of 10%.

a) Represent graphically the budget line of the family. Find the optimal level of expenditures in each period.
b) Find the function for the demand for credit by Mr. Ferreira. How does the demand for credit change with \( M_1 \), \( M_2 \) and the interest rate?
c) Find the savings function. What relation does it have with the demand for credit?
d) Find the interest rate that makes Mr. Ferreira not to borrow from Mr. Lima?
e) Currently, Mr. Lima has some financial problems and, thus, the maximum amount he can lend is 10. Find the optimum for Mr. Ferreira in these new circumstances.
f) Find the subjective rate for intertemporal preference in the optimum. Compare it with the interest rate.
g) What is the interest rate that, without credit restrictions, would induce a credit of 10? Compare it with the previous question.

Exercise 2
For a certain interest rate, a particular individual decides to borrow. Can one say that if the interest rate increases he would be in a worse situation?

Exercise 3
A certain individual acts in a perfect capital market. He is considering the hypothesis of accepting a project with an initial investment of \( X \), now, and a return of \( Z \), in the next period. Can the decision be different if his intertemporal preferences change?

Exercise 4
A certain consumer decides the amount of expenditures in consumption goods in periods 1 and 2, \( C_1 \) and \( C_2 \) respectively, using the knowledge of the incomes in those periods, \( Y_1 \) and \( Y_2 \) respectively, and the interest rate. Consider an active interest rate, \( r_a \), higher than the passive \( r_b \).

Represent graphically the consumer’s budget line.

Assuming that the consumer has strictly convex to the origin preferences, identify the optimal points to which the subjective rate of intertemporal preference is different from any of the interest rates. Justify.
3.3 Revealed Preference

Exercise 1
Analyze the transitivity of the consumer’s preferences observed in the following three situations:

<table>
<thead>
<tr>
<th>Situation A</th>
<th>Situation B</th>
<th>Situation C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>1800</td>
<td>2000</td>
</tr>
<tr>
<td>Price of x</td>
<td>200</td>
<td>400</td>
</tr>
<tr>
<td>Price of y</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Quantity of x</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Quantity of y</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

Exercise 2
A certain consumer can only buy goods x and y. The price of y is €1. Discuss the consistency of the consumer’s behavior in situations A and B:

a) Situation Income Price of x Quantity of x
   A  40  1  20
   B  60  2  25

b) Situation Income Price of x Quantity of x
   A  40  1  20
   B  60  2  15

Exercise 3
A certain consumer spends all his income in the consumption of goods x and y. Even though it is not possible to derive a mathematical expression for the consumer’s preferences, his consumption decisions were observed in situations A and B:

<table>
<thead>
<tr>
<th>Situation A</th>
<th>Situation B</th>
</tr>
</thead>
<tbody>
<tr>
<td>x*</td>
<td>8</td>
</tr>
<tr>
<td>y*</td>
<td>6</td>
</tr>
<tr>
<td>M</td>
<td>20</td>
</tr>
<tr>
<td>Py</td>
<td>2</td>
</tr>
<tr>
<td>Px</td>
<td>1</td>
</tr>
</tbody>
</table>
a) Using the graphical analysis, check if the consumer’s behavior is consistent with the Weak Axiom of the Revealed Preference (WARP). Which of the 2 situations is better to the consumer?

b) Consider another situation, C, with m = €20, Px = €2 and Py = €2. Using the graphical analysis of the previous question, explain carefully in which of the segments of the new budget restriction will the optimal bundle be if:
   i) x is a Giffen good
   ii) x and y are normal goods; x and y are substitutes
   iii) y is an inferior good
   iv) good x has inelastic demand
   v) y is a luxury good and x is a necessary good

(Hint: remember that if the income offer curve is linear and cross the origin, then the income elasticity of demand is unitary)

c) Consider, now, a situation D such that m = €22, Px = €1 and Py = €2. Using the information in a), find the segment of this new budget restriction where the consumer’s choice is consistent with the maximizing utility model.

Exercise 4

Mr. Revealed spends all his income in the consumption of goods x and y. No mathematical expression is known for his preferences. The behavior of Mr. Revealed was observed in three alternative situations (1, 2 and 3) where he chose bundles A, B and C, respectively.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Expenditure</th>
<th>Px</th>
<th>Py</th>
<th>Chosen Bundle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>20</td>
<td>1</td>
<td>1</td>
<td>X 5, Y 15</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
<td>?</td>
<td>1</td>
<td>5, 0</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>4</td>
<td>?</td>
<td>2.5, 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Complete the table and justify the values found.

b) Do Mr. Revealed's decisions verify the Strong Axiom of Revealed Preference (SARP)? Justify carefully.

c) Consider a situation with m = €50, Px = €2 and Py = €1. What set of bundles can Mr. Revealed buy in order to be consistent with the Strong Axiom of Revealed Preference (SARP).

d) Consider, now, a consumer that spends all his income in goods 1 and 2 which cost €1 each. This consumer, who as homothetic preferences, chose the bundle X1=12 and X2=36 in a certain situation.

Assume that this consumer was given an endowment of x1 units of good 1. Graphically show the relation between the number of units of good 1 consumed X1 (y-axis) and the endowment x1 (x-axis).

(Hint: Consider x1<32 and start with the representation of the consumer’s income offer curve)
3.4 Welfare Measure

Exercise 1
Consider a consumer with Cobb-Douglas preferences described by the following expenditure and ordinary demand functions:

\[ M = 5 U \left( \frac{p_1}{2} \right)^{0.4} \left( \frac{p_2}{3} \right)^{0.6} \]

\[ x_1 = \frac{2M}{5p_1} \]

\[ x_2 = \frac{3M}{5p_2} \]

Initially, the consumer has an income of €100 and prices are €10 for both good 1 and 2.

a) Suppose that the price of good 1 increases by 100%. Find, analytically and graphically, the compensating and equivalent variations and the consumer's surplus variation. Interpret and compare.

b) Answer the previous question again considering now a decrease in the price of good 1 to €1.

Exercise 2
When summer begins the danger of fire in the Flaming Mountain, a natural park, increases. Consequently, entry in the park is carefully controlled and a payment of €5 is demanded. The Mayor complains about the scarce financial resources for prevention and fire fight. Thus, he proposes an increase in the entry tariff to €10.

The following relations describe the preferences of a representative inhabitant between visits to the park (x) and other goods (y):

\[ U = (x,y)^{0.5} \]

\[ M = 2U \left( p_x \cdot p_y \right)^{0.5} \]

\[ x = \frac{M}{2p_x} \]

\[ y = \frac{M}{2p_y} \]

The representative inhabitant has an income of €100 and the price of the other goods is 1.

a) How much is the representative inhabitant willing to pay in order to avoid the increase in the entry tariff? Which welfare measure underlies this reasoning? Justify.

b) The Mayor increases the price to €10. What is the minimum amount the representative inhabitant must receive in order to keep the initial number of visits to the park? How do you call the value found? Justify.

Exercise 3
Consider a consumer with quasi-linear preferences described by the following relations:

\[ M = p_2 \left( U - \ln \left( \frac{p_2}{p_1} \right) + 1 \right) \]

\[ x_1 = \frac{p_2}{p_1} \]

\[ x_2 = \frac{M}{p_2} - 1 \]

His available income to spend in the goods 1 and 2 is €100 and the prices are \( p_1 = 5 \) e \( p_2 = 10 \).

a) There are some rumors about a decrease in the price of good 1 to €1. What is the minimal amount to be given to the consumer so that he would be willing to give up this price decrease? If the consumer would still want to have access to the new bundle, even though without the price decrease, what amount would he receive as a subsidy? Define both monetary variations and represent them graphically.
b) Effectively, the price of good 1 decreases to €1. How much is the consumer willing to pay in order to assure that the price decrease really takes place? What tax amount must be collected so that, at the new prices, the consumer would keep the initial bundle? Define the monetary variations found and represent them graphically.

c) Calculate the consumer’s surplus variation caused by the price decrease and represent it graphically.

d) Compare and discuss the previous questions’ results.

Exercise 4

Answer the previous questions considering a consumer with preferences given by

\[ U = \min(x_1, x_2) \]

and with expenditures and ordinary demand functions:

\[ M = U(p_1 + p_2) \]

\[ x_1 = x_2 = \frac{M}{p_1 + p_2} . \]

* Exercise 5

A housewife buys 2 kilo of meat a week. Assume that the price of the meat increases €0.50 per kilo. Verify:

a) The compensating variation associated to this change in the price must be less or equal to €1.

b) The equivalent variation associated to this change in the price must be at least €1.

Exercise 6

a) Is the consumer better or worse if the price of a certain good increases, while other prices and income remain constant? Can he be just the same? If yes, in which case?

b) What is the only situation in which a consumer is indifferent to an income increase?

c) A certain consumer chooses \( x_1 = 4 \) and \( x_2 = 2 \) if \( p_1 = 1, p_2 = 2 \) and \( m = 8 \). Currently, \( p_1 = 2, p_2 = 4 \) and \( m = 16 \) and each component of the chosen bundle increased 100%. If the previous choice was optimal, can this last choice also be optimal?

d) Given that \( p_1 = 1 \), one know that, for a consumer to keep his utility level when \( p_2 \) increases 2 units, \( p_1 \) must decrease 0.5. If \( p_1 = 10 \) and \( p_2 \) increases from 2 to 4, what can you say about the necessary increase in the price of good 1 such that the consumer keeps his utility level?

Exercise 7

In a Portuguese region, where the population works mainly in agricultural activities, the drought in the last three years caused an increase in the pollution level of the only well in the region. This well belongs to the city hall and recently it has set up equipment for water treatment.

In order to finance the equipment, the Mayor decided an increase in the price of water from €1 to €1.5 per cubic meter. Nevertheless, an independent councilor did not agree and publicly defended an increase in the monthly tariff the farmers must pay in order to have access to the well, keeping the price of water in €1 per cubic meter.

a) From the farmers’ point of view, who consume water for irrigation (A) and other purposes (C), is the independent councilor right?

b) The Regional Agricultural Cooperative contracted your services as counselor and gave you the following information:
- the representative farmer has compensated demand functions given by:
  \[ A = 2,884. \, U \cdot (P_d/P_a)^{0.82} \quad \text{and} \quad C = 13,138. \, U \cdot (P_d/P_c)^{0.18}; \]
- the monthly income is €110;
- the price of C is 1;
- before the decision the farmers’ utility index was \( U_0 = 6,241 \) and after the increase in the
  price of the water it changed to \( U_1 = 5,802. \)
- the current monthly tariff to have access to the well if €10.

They want to know if they may believe in the independent councilor’s idea of increasing the
monthly tariff in €5.

Indifferent to the critics, the Mayor kept the price of water in €1.5 per cubic meter. However,
soon there will be elections and the independent councilor is a strong candidate to be the next
Mayor. He promised to decrease the well’s tariff such that the farmers would be able to use
the same quantity of water they were using before the increase in the price of water. The current
Mayor, who wants to be re-elected, was forced to change his position and promises to decrease
the well’s tariff in €5.

c) The farmers want you to tell them if the candidates’ promises totally compensate them and
in which of them to vote. (Assume that the candidates are equal in everything else except in
this subject.)

d) After the elections, and before the old Mayor leaves the City Hall, he asks you to find the
ordinary demand function of water such that in the future it can be used in the decisions.
Can you do it?