Results from Welfare Economics - 1

- General equilibrium in Edgeworth boxes: consumption with endowments, 2 consumers, 2 goods.

- **1st Welfare Theorem:** The allocation of resources that results from a competitive equilibrium is Pareto Optimal.

- **2nd Welfare Theorem:** If preferences and technology are convex given a Pareto optimal resource allocation we can find a competitive equilibrium that generates that resource allocation. That equilibrium is obtained by a suitable distribution of the initial endowments in the economy.
Transfer of good y from consumer 1 to consumer 2

Transfer of good x from consumer 1 to consumer 2

In this example the redistribution from consumer 1 to 2, from consumption vector $\alpha$ to consumption vector $\beta$, corresponds to a move along the utility possibility frontier.

Endowment transfers correspond to “lump-sum” taxes.

Results from Welfare Economics - 2

Is it the case that the 2nd Welfare Theorem implies that there is no tradeoff between efficiency and equity? Can we do any redistribution without compromising efficiency?

No, There are endowments that cannot be redistributed. Ex: time, intelligence, creativity, energy, physical abilities and characteristics (highly valued by the market), etc.

Political decision makers have to choose between efficiency and equity. Their “values” can be analytically represented by Social Welfare Functions- SWFs.

Results from Welfare Economics - 3
Negishi’s Theorem. Consider a Social welfare function of the form
\[ \text{SWF} = \sum \lambda_i U_i. \]

Then, the allocation and utility distribution that maximizes this SWF is Pareto optimal for positive \( \lambda \)'s. Moreover, if the utility possibility frontier is convex, for any allocation \( A \) and utility distribution \( U \) that is Pareto optimal there is a vector of welfare weights \( \lambda \)'s such that \( A \) and \( U \) are the solution to the SWF maximization problem.

Fair Allocations 1

- SWF are defined over the space of individual utilities and they summarize ethical judgments and moral values. But the implications of these judgments are sometimes difficult to evaluate.
- Also it is interesting to examine issues of fairness directly in the space of goods (allocations).
- If we have a total endowment in the economy of goods \( x \) and \( y \) given by \((X, Y)\) and \( n \) individuals equally deserving, most would agree that giving each individual a vector \((X/n, Y/n)\) is a fair allocation. Intuitively, there is an ethical appeal to symmetry.
- But is the equal division allocation efficient? NOT NECESSARILY because people have different tastes.
In this case, the symmetric allocation $\omega_e$ is outside the contract curve and thus is Pareto inefficient. This example shows symmetry is not a good criterion for judging an allocation.

Naturally, we can improve on $\omega_e$ if we allow for trade among agents. Are all allocations obtained from trading starting at an equal division equitable? NOT NECESSARILY. If $n=3$ and persons 1 and 2 have the same preferences whereas 3 is different, and 1 and 3 trade and make themselves better off, then 2 will be worse off than 3 and evaluate the post trade allocation as unfair.

How can we define an equitable allocation? By a NO ENVY condition. An allocation is equitable if no agent prefers another agent’s bundle of goods to his own.

In an Edgeworth box that means an agent does not prefer to swap allocations with the other agent.

Allocation $\omega_i$ is envy free or equitable since no one would like to trade places. But it is not efficient.
We know the set of all efficient allocations is the contract curve. What is the set of all envy free allocations in an Edgeworth box?

We first build the envy boundary for agent 1, then for agent 2 and then take the intersection.
Fair Allocations 6

- The set of FAIR allocations is the intersection of the sets of efficient allocations, the contract curve, and the set of all envy free allocations.

Fair Allocations 7

One way to find fair allocations is to look for a competitive equilibrium started from an equal division of the endowments. In this case no agent envies other agents.
The fact that a competitive equilibrium from equal division is a fair allocation tells us that the market is not only efficient but also fair provided endowments are equitably allocated to begin with.

This may be interpreted as saying the market is fair as a process.

Unfortunately, there are limits to the significance of the result. In production economies envy-free and efficient allocations may not exist – we go back to the problem that some endowments simply cannot be redistributed.

Preferences: $U^1 = \ln x_1 + 2\ln y_1$ $U^2 = 2\ln x_2 + \ln y_2$

Endowments: $X = 20$ $Y = 10$

The set of Fair allocations has all efficient and envy-free allocations.

Contract curve $MRS^1 = MRS^2 \iff \frac{y^1}{2x^1} = \frac{y^2}{x^2} \iff \frac{y^1}{2x^1} = \frac{10-y^1}{20-x^1}$: solving for $y^1$ we find $y^1 = \frac{40x^1}{20+3x^1}$.

Points in the contract curve: $(x^1=0, y^1=0), (x^1=4, y^1=5), (x^1=10, y^1=8), (x^1=20, y^1=10)$. 

Contract Curve
Fair Allocations—Cobb-Douglas Example 2

- The envy free frontier for agent 1 is defined by: \( U^1 = \ln x_1 + 2\ln y_1 = \ln(20-x_1) + 2\ln(10-y_1) \iff x_1 = 20(10-y_1)^2/(y_1^2+(10-y_1)^2) \)
- The frontier for agent 2 is \( U^2 = 2\ln x_2 + \ln y_2 = 2\ln(20-x_2) + \ln(10-y_2) \), \( y_2 = 10(20-x_2)^2/(x_2^2+(20-x_2)^2) \). For the purpose of graphical representation we can re-write 2’s frontier in terms of \( x_1 \) and \( y_1 \):
  \[ 10-y_1 = 10(x_1)^2/(x_1^2+(20-x_1)^2) \iff y_1 = 10(20-x_1)/((x_1^2+(20-x_1)^2) \]

Fair Allocations—Cobb-Douglas Example 3

- Fair allocations are a subset of the contract curve \( y_1 = 40 x_1/(20+3x_1) \). The intersection at the envy frontier 1, \( x_1 = 20(10-y_1)^2/(y_1^2+(10-y_1)^2) \), gives us \( y_1 = 6.14 \) and \( x_1 = 6.39 \).
- 2’s frontier, in terms of \( x_1 \) and \( y_1 \), is \( y_1 = 10(20-x_1)^2/(x_1^2+(20-x_1)^2) \). The intersection is \( y_1 = 7.16 \) and \( x_1 = 7.73 \).
Fair Allocations—Cobb-Douglas Example 4

- Finally, let us check that the competitive equilibrium from equal division is Fair. There are two goods but by Walras’ Law we only need to calculate the equilibrium in one market. Normalize the price of $x$ to 1 (only relative prices matter).
- The consumer’s problem is
  \[ \text{Max}_{x,y} \ L = \ln x + 2\ln y + \lambda [10-x-x-y] \].

Solving the First order conditions,
\[ \frac{\partial L}{\partial x} = \frac{1}{x} - \lambda = 0; \quad \frac{\partial L}{\partial y} = \frac{2}{y} - \lambda [10 - x] + p(5 - y) = 0 \]
the demand for $y$ is given by $y^d = 20/3p + 10/3$.
- The demand for $y$ from consumer two is $y^d_2 = 10/3p + 5/3$.
- In the general equilibrium demand equals supply: $10 = 10/3p + 5/3 + 20/3p + 10/3$, $p=2$. This implies $y^*_1 = 20/3 = 6.66$ and $y^*_2 = 10/3 = 3.33$. This also means $x^*_1 = 6.66$ and $x^*_2 = 13.33$.

Fair Allocations—Cobb-Douglas Example 5

![Fair Allocations](image)
Assumptions of perfect competition

- Agents are price takers.
  - Many agents on the demand and on the supply side. Each agent is small. Market failures resulting from violation: monopoly (natural or not...), monopsony, cartels, etc.

- Resources are privately owned and used
  - Market failures resulting from violation: public goods, externalities

- No restrictions to the mobility of resources
  - Market failures resulting from violation in the form of barriers to entry and exit in certain professions or industries (sectors).

- All agents have equal access to relevant information
  - Market failures resulting from violation: asymmetric information problems such as moral hazard or adverse selection.

Welfare Measures - 1

- CS neglects income effects. It is an adequate measure for situations where the income effect is negligible, typically when the good has a small household budget share.
- To measure changes in some important markets such as the labor market, housing and other large household budget share goods we should use exact measures: the equivalent and the compensatory variations.
Welfare Measures - 2

- Concepts
  - $U(x, y)$ utility function; $M$ Income; $p_x, p_y$ prices
  - $X(M, p_x, p_y)$, $Y(M, p_x, p_y)$ demand functions
  - $U(X(M, p_x, p_y), Y(M, p_x, p_y)) = V(M, p_x, p_y)$ indirect utility
  - $V(M, p_x, p_y) = U^0 \Leftrightarrow E(U^0, p_x, p_y) = M$, expenditure function

- Compensating Variation. If price vector changes $P^0 \rightarrow P^1$
  - Definition: $V(M+CV, P^1) = V(M, P^0)$
  - $CV(U^0, P^0, P^1) = E(U^0, P^1) - E(U^0, P^0)$

- Equivalent Variation. If price vector changes: $P^0 \rightarrow P^1$
  - Definition: $V(M-EV, P^0) = V(M, P^1)$
  - $EV(U^1, P^0, P^1) = E(U^1, P^1) - E(U^1, P^0)$

Results from Welfare Economics - 5

- $EV\text{ e }CV$ can be graphically estimated by using compensated (Hicksian) demands.

![Diagram showing welfare measures]

What is the sign of the income elasticity assumed? How do we represent $CV\text{ e }EV$ in the goods $(x,y)$ space?